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# ON ALMOST-PRIMES IN ARITHMETIC PROGRESSIONS

By

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## 1. Introdnction.

Let $P_r$ denote integers with at most $r$ prime factors counted according to multiplicity. In 1975, Y, Motohashi [9] showed that there exists a $P_3$ such that

$$P_3 \equiv a \pmod{q}, \qquad P_3 \ll q(\log q)^{70}$$

for any fixed non-zero integer $a$ and almost-all moduli $q$ with $(q, a)=1$. His argument based upon the weighted linear sieve and the Brun-Titchmarsh theorem on average, which are due to H. E. Richert [10] and C. Hooley [4], respectively. H. Iwaniec's fundamental works [6, 7], therefore, suggest possibilities of an improvement upon the above result. In this paper we present an estimation for $P_2$. We shall prove the following

THEOREM. *Let $Q$ be a large parameter and $a$ be any fixed integer, $0<|a| \leqq Q$. Then, except possibly for $O(Q/\log Q)$ moduli $q$ with $(q, a)=1$ and $Q<q\leqq 2Q$, there exists a $P_2$ such that*

$$P_2 \equiv a \pmod{q}, \qquad P_2 \leqq \tau(a) q(\log q)^7$$

*where the implied $O$-constant is absolute and $\tau$ denotes the divisor function.*

Our proof of Theorem is performed by a simple modification of the argument in our previous paper [8], in which the dual problem is considered. In fact, the numerical work in the main term from sieve estimate is identical. Succeeding to Hooley's investigation [4] we treat the remainder terms with the same manner as in [8]. Our main lemma (see Lemma 1 below) is weaker than E. Fouvry's works [1-3] in its scope; however, it will be found to be suitable for an application to the weighted sieve.

We use the standard notation in number theory. Especially, $\bar{r}$, used in either $r/s$ or congruence $(\bmod q)$, means that $\bar{r}r \equiv 1 \pmod{s}$. $\sum_{x=1}^{y}{}^{*}$ stands for the summation with restriction $(x, y)=1$. $n \sim N$ means $N \leqq N_1 < n \leqq N_2 \leqq 2N$ for some $N_1$ and $N_2$. $\varepsilon$ denotes a small positive constant and the constants implied in the

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