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COMPACTNESS CRITERIA FOR RIEMANNIAN MANIFOLDS WITH COMPACT UNSTABLE MINIMAL HYPERSURFACES

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1. Introduction

In this paper, we shall prove the following Theorem.

THEOREM A. *Let N be a complete Riemannian manifold with a compact embedded unstable minimal hypersurface M. Suppose that there exists a positive constant s_0 such that along each unit speed geodesic $\gamma: [0, \infty)\to N$ emanating from each point in the tubular neighborhood $U_{s_0}(M):=\{\wp\in N; \mathrm{dist}_N(\wp, M)<s_0\}$ the Ricci curvature satisfies*

$$\liminf_{r\to\infty}\int_0^r \mathrm{Ric}_N(d\gamma/dt, d\gamma/dt)dt\geqq 0.$$

Then N is compact.

The Myers' theorem [11] is one of the most well-known results relating the curvature and the topology of a complete Riemannian manifold N, which states that if the Ricci curvature has a positive lower bound then N is compact. In [1], Ambrose proved a generalization of Myers' theorem, that is, if there is a point $\wp\in N$ such that along each unit speed geodesic $\gamma: [0, \infty)\to N$ emanating from $\wp$ the Ricci curvature satisfies

$$\int_0^\infty \mathrm{Ric}_N(d\gamma/dt, d\gamma/dt)dt=+\infty$$

then N is compact. It should be pointed out that in this result the Ricci curvature is not required to be everywhere nonnegative. Further developments can be found in Galloway [9] and different sorts of extensions of Myers' theorem can be found in Avez [3], Calabi [5] and Shiohama [12].

Theorem A is an Ambrose-type theorem for Riemannian manifolds with compact embedded unstable hypersurfaces (see also Remark in section 3). It should be also pointed out that in Theorem A the existence of the global unit normal vector field on M is not required.

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