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SOME INEQUALITIES ON $|\nabla R|$, $|\nabla \mathrm{Ric}|$ AND $|dr|$ IN RIEMANNIAN MANIFOLDS

By

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Introduction. Let M^n be an $n(>1)$ dimensional Riemannian manifold. We denote by $g=(g_{ji})$, $R=(R_{kji}{}^{h})$, $\mathrm{Ric}=(R_{ji})=(R_{rji}{}^{r})$ and $r=(R_i{}^i)=(R_{ji}g^{ji})$ the metric tensor, the curvature tensor, the Ricci tensor and the scalar curvature respectively. ∇ means the operator of the covariant differential, and we put ${}^c\nabla R=(\nabla_r R_{kji}{}^{r})$. The purpose of this paper is to give some inequalities which hold among the norms of ∇R, ${}^c\nabla R$ and dr. Though we do not know whether such inequalities are worthy to be studied or not, the cases when the equalities hold in our inequalities seem meaningful.

§ **1** will be devoted itself to preliminaries. Denoting the norm of a tensor T by $|T|$, we shall show in § **2** two inequalities among $|\nabla R|$, $|{}^c\nabla R|$ and $|dr|$. In one of the inequalities the equality holds if and only if the manifold has harmonic Weyl tensor. An application will be given. In § **3** an inequality which holds between $|\nabla \mathrm{Ric}|$ and $|dr|$ will be proved. In § **4** we shall give among $|\nabla R|$, $|{}^c\nabla R|$ and $|dr|$ two inequalities which are different from those in § **2**. An inequality for the Codazzi tensor will be shown in § **5**, and in the last section $|\nabla \mathrm{Ric}|$ in Kaehlerian manifolds will be discussed.

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§ **1. Preliminaries.** Let M^n be an n dimensional Riemannian manifold. We follow the notations in Introduction. Tensors are represented by their components with respect to the natural basis, unless otherwise stated, and the summation convention is assumed. ∇ denotes the operator of covariant differential. We have $\nabla R=(\nabla_l R_{kji}{}^{h})$.

Let us put

$$ {}^c\nabla R=(S_{kji}), $$

where

$$ S_{kji}=\nabla_r R_{kji}{}^{r}. $$

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