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ON THE NEUMANN PROBLEM FOR SOME LINEAR HYPERBOLIC SYSTEMS OF 2ND ORDER WITH COEFFICIENTS IN SOBOLEV SPACES

By

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Introduction.

Let Ω be a domain in an n-dimensional Euclidean space $\boldsymbol{R}^n$, its boundary Γ being a C^∞ and compact hypersurface. Throughout the present paper, we assume that $n\geqq 2$[(1)]. Let $x=(x_1, \cdots, x_n)$ denote points of $\boldsymbol{R}^n$ and t a time variable. For differentiations we use the symbols: $\partial_t=\partial_0=\partial/\partial t$ and $\partial_j=\partial/\partial x_j$ $(j=1, \cdots, n)$. In this paper, we consider the following mixed problem:

$$
\text{(N)}\quad\begin{cases} P(t)[\vec{u}(t)]=\partial_t^2\vec{u}(t)-\partial_i(A^{i0}(t)\partial_t\vec{u}(t)+A^{ij}(t)\partial_j\vec{u}(t))=\vec{f}_\Omega(t) & \text{in } (0, T)\times\Omega, \\ Q(t)[\vec{u}(t)]=\nu_i A^{ij}(t)\partial_j\vec{u}(t)+B^j(t)\partial_j\vec{u}(t)+B^0(t)\partial_t\vec{u}(t)=\vec{f}_\Gamma(t) & \text{on } (0, T)\times\Gamma, \\ \vec{u}(0)=\vec{u}_0, \qquad \partial_t\vec{u}(0)=\vec{u}_1 & \text{in } \Omega, \end{cases}
$$

where T is a positive constant and $\vec{u}={}^t(u_1, \cdots, u_m)$ (=the row vector of length m and ${}^tM$ means the transposed vector (resp. matrix) of the vector (resp. matrix) M). Here and hereafter, the summation convention is understood such as the sub and superscripts i, i', j, j' (resp. p, q) take all values 1 to n (resp. 1 to $n-1$). For any vector valued function $\vec{u}={}^t(u_1, \cdots, u_m)$, we put $\partial_t^j\partial_x^\alpha\vec{u}={}^t(\partial_t^j\partial_x^\alpha u_i$ $\cdots, \partial_t^j\partial_x^\alpha u_m)$. The $\nu_i=\nu_i(x)$ are real-valued functions in $C_0^\infty(\boldsymbol{R}^n)$ such that the vector $\nu(x)=(\nu_1(x), \cdots, \nu_n(x))$ represents the unit outer normal to Γ at $x\in\Gamma$. In the present paper, functions are assumed to be real-valued, unless ortherwise specified. Below, I will always refer to the closed interval containing $[0, T]$ strictly, say, $I=[-\tau, T+\tau]$ $(\tau>0)$. And also, K will always refer to the fixed integer $\geqq[n/2]+2$, which represents the order of regularitiy of solutions and coefficients of the operators $P(t)$ and $Q(t)$. The $A^{il}(t)=A^{il}(t, x)$ and $B^l(t)=B^l(t, x)$ $(l=0, 1, \cdots, n; i=1, \cdots, n)$ are $m\times m$ matrices of functions satisfying the

(1) When $n=1$, excepting the notations, we can treat the same problem without essential change. However, for the notational simplicity, we shall only treat the case where $n\geqq 2$, below.

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