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EXPONENTIAL AND SUPER-EXPONENTIAL LOCALIZATIONS FOR ONE-DIMENSIONAL SCHRÖDINGER OPERATORS WITH LÉVY NOISE POTENTIALS

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§ 0. Introduction.

Let us consider the following random Schrödinger operator in $L^2(\boldsymbol{R}; dt)$:

$$H_\omega = -d^2/dt^2 + Q'_\omega(t), \tag{0-1}$$

where $\{Q'_\omega(t); -\infty<t<+\infty\}$ is a temporally homogeneous Lévy process and $Q'_\omega(t)$ is the "derivative" of its sample function. Intuitively speaking, $\{Q'_\omega(t)\}_{t\in\boldsymbol{R}}$ is a continuous parameter family of i. i. d. random variables, which we will call "Lévy noise", so that the above H_ω can be viewed as an idealization of the Schrödinger operator with random potential, and it may be of some interest to analyze in detail such an idealized model of disordered system.

On the other hand, it is well known that almost every sample function of a Lévy process is not differentiable (except the case of a trivial Lévy process $Q_\omega(t)=ct$, with a real constant c). Hence the expression (0-1) has only a symbolical meaning. The precise definition of H_ω was given by the present author ([25]), and it was shown that H_ω can actually be realized as a random self-adjoint operator in $L^2(\boldsymbol{R}; dt)$. Moreover, the exact location of the spectrum of H_ω was determined.

The purpose of this paper is to study the properties of spectrum and eigenfunctions of H_ω in more detail than in [25]. It will be shown that under some condition on $\{Q_\omega(t)\}$, almost every realization of H_ω has pure point spectrum with exponentially decaying eigenfunctions (exponential localization—see Theorem 5). A remarkable fact is that in some other cases, the eigenfunctions decay faster than exponentially (Theorem 6). We would like to refer to this phenomenon as "super-exponential localization". Moreover, it will be shown that under some conditions on the Lévy measure of $\{Q_\omega(t)\}$, the eigenfunctions, in a rough sense, behave like $\exp[-|t|^\alpha]$ with $\alpha>1$, or even like $\exp[-\exp[\exp[\cdots\exp[|t|^\alpha]\cdots]]$ with $\alpha>0$, as $|t|\to\infty$ (Theorem 7).

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