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COMPLETE RIEMANNIAN MANIFOLD MINIMALLY IMMERSED IN A UNIT SPHERE $S^{n+p}(1)$

By

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1. Innroduction.

Let M^n be an n-dimensional Riemannian manifold which is minimally immersed in a unit sphere $S^{n+p}(1)$ of dimension $n+p$. If M^n is compact, then many authors studied them and obtained many beautiful results (for examples [1], [3], [4], [5] and [6]). In this paper, we make use of Yau's *maximum principle* to extend these results to complete manifolds with Ricci curvature bounded from below.

2. Preliminaries.

Let M^n be an n-dimensional Riemannian manifold which is minimally immersed in a unit sphere $S^{n+p}(1)$ of dimension $n+p$. Then the *second fundamental form* h of the immersion is given by $h(X, Y)=\tilde{\nabla}_X Y-\nabla_X Y$ and it satisfies $h(X, Y)=h(Y, X)$, where $\tilde{\nabla}$ and ∇ denote the covariant differentiation on $S^{n+p}(1)$ and M^n respectively, X and Y are vector fields on M^n. We choose a local field of orthonormal frames $e_1, \cdots, e_n, \cdots, e_{n+p}$ in $S^{n+p}(1)$ such that, restricted to M^n, the vector $e_1, \cdots, e_n$ are tangent to M^n. We use the following convention on the range of indices unless otherwise stated: $A, B, C, \cdots=1, 2, \cdots, n+p$; $i, j, k, \cdots=1, 2, \cdots, n$; $\alpha, \beta, \cdots=n+1, \cdots, n+p$. And we agree that repeated indices under a summation sign without indication are summed over the respective range. With respect to the frame field of $S^{n+p}(1)$ chosen above, let $\tilde{\omega}_1, \cdots, \tilde{\omega}_{n+p}$ be the dual frames. Then structure equations of $S^{n+p}(1)$ are given by

$$d\tilde{\omega}_A=\sum\tilde{\omega}_{AB}\wedge\tilde{\omega}_B, \quad \tilde{\omega}_{AB}+\tilde{\omega}_{BA}=0, \tag{2.1}$$

$$d\tilde{\omega}_{AB}=\sum\tilde{\omega}_{AC}\wedge\tilde{\omega}_{CB}-\tilde{\omega}_A\wedge\tilde{\omega}_B. \tag{2.2}$$

Restricting these forms to M^n, we have the structure equations of the immersion:

$$\omega_\alpha=0, \tag{2.3}$$

$$\omega_{i\alpha}=\sum h_{ij}^\alpha\omega_j, \quad h_{ij}^\alpha=h_{ji}^\alpha, \tag{2.4}$$

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