DOMESTIC TRIVIAL EXTENSIONS OF SIMPLY CONNECTED ALGEBRAS

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Dedicated to Professor H. Tachikawa on his 60th birthday

Abstract Let A be a finite-dimensional, basic and connected algebra (associative, with 1) over an algebraically closed field. It is called simply connected it it is triangular and, for any presentation of A as a bound quiver algebra, the fundamental group of its bound quiver is trivial. Let T(A) denote the trivial extension of A by its minimal injective cogenerator. We show that, if A is simply connected, then the following conditions are equivalent: (i) T(A) is representation-infinite and domestic, (ii) T(A) is 2-parametric, (iii) there exists a representation-infinite tilted algebra B of Euclidean type \tilde{D}_n or \tilde{E}_p such that $T(A) \cong T(B)$, (iv) A is an iterated tilted algebra of type \tilde{D}_n or \tilde{E}_p .

Introduction.

Let k denote a fixed algebraically closed field, and A a finite-dimensional k-algebra (associative, with an identity) which we shall moreover assume to be basic and connected. We shall denote by mod A the category of finite-dimensional right A-modules. The $trivial\ extension\ T(A)$ of A by its minimal injective cogenerator bimodule $DA = \operatorname{Hom}_k(A, k)$ is the algebra whose additive structure is that of the group $A \oplus DA$, and whose multiplication is defined by:

$$(a, f)(b, g) = (ab, ag+fb)$$

for $a, b \in A$ and $f, g \in A(DA)_A$. Then T(A) is a self-injective and, in fact, a symmetric algebra.

Trivial extension algebras have been extensively investigated in representation theory. First, in the representation-finite case, they were studied by Müller [32], Green and Reiten [22] and Iwanaga and Wakamatsu [30] when the radical

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