

A NOTE ON REAL HYPERSURFACES OF A COMPLEX HYPERBOLIC SPACE

By

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Introduction.

A Kaehlerian manifold of constant holomorphic sectional curvature c is called a complex space form. The complete and simply connected complex space form of complex dimension n consists of a complex projective space $P^n C$, a complex Euclidean space C^n or a complex hyperbolic space $H^n C$, according as $c > 0$, $c = 0$ or $c < 0$.

Many subjects for real hypersurfaces of a complex projective space $P^n C$ have been studied [1], [4], [5] and [6]. One of which, done by Kimura [6], asserts the following interesting result.

THEOREM K. *There are no real hypersurfaces of $P^n C$ with parallel Ricci tensor on which $J\xi$ is principal, where ξ denotes the unit normal and J is the complex structure of $P^n C$.*

A Riemannian curvature of a Riemannian manifold M is said to be *harmonic* if the Ricci tensor S satisfies the Codazzi equation, that is,

$$(0.1) \quad \nabla_x S(Y, Z) - \nabla_y S(X, Z) = 0$$

for any tangent vector fields X, Y and Z , where ∇ denotes the Riemannian connection of M . This condition is essentially weaker than that of the parallel Ricci tensor [2]. From this point of view, Kwon and Nakagawa [5] extends recently the following:

THEOREM K-N. *There are no real hypersurfaces with harmonic curvature of $P^n C$ on which $J\xi$ is principal.*

Now we are interested in these problems in the case of $c < 0$, that is, the ambient space is a complex hyperbolic space $H^n C$. Montiel [7] stated that there are no Einstein real hypersurfaces in $H^n C$, and classified the pseudo-Einstein real hypersurfaces of $H^n C$. In this paper, we will prove