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A NOTE ON THE PROJECTIVE NORMALITY OF SPECIAL LINE BUNDLES ON ABELIAN VARIETIES

By

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Dedicated to Professor Yukihiro Kodama on his 60th birthday

Introduction.

Let L be an ample line bundle on an abelian variety A of dimension g defined over an algebraically closed field k. It is well known that $L^{\otimes 2}$ is base point free and $L^{\otimes 3}$ is very ample and projectively normal. Moreover we know that

$$\Gamma(A, L^{\otimes a})\otimes\Gamma(A, L^{\otimes b}) \longrightarrow \Gamma(A, L^{\otimes a+b})$$

is surjective if $a\geqq 2$ and $b\geqq 3$ (Koizumi [3], Sekiguchi [8], [9]). But in the case of $a=b=2$, this map is not surjective in general. In this paper we determine the condition of projective normality of $L^{\otimes 2}$ for some ample line bundle L. Our result is as follows.

THEOREM. *If L is a symmetric ample line bundle of separable type, $l(A, L)$ is odd and assume that* char$(k)\neq 2$, *then $L^{\otimes 2}$ is projectively normal if and only if $Bs|L|\cap A[2]=\phi$.*

In §1 we prove the above theorem for abelian varieties defined over $\boldsymbol{C}$. In §2 we give the Mumford's theory of a theta group (Mumford [4], [5]). In §3 we prove the above theorem in general by the theory in §2.

Notations.

char(k): The characteristic of a field k

f^*: The pull back defined by a morphism f

$\underline{L}$: The invertible sheaf associated to a line bundle L

$\mathcal{O}_A$: The invertible sheaf of a variety A

(L^g): The self intersection number

$|L|$: The set of all effective Cartier divisors which define a line bundle L

$Bs|L|$: The set defined by $\bigcap_{D\in|L|} D$

$\Gamma(A, L)$: The set of global sections of a line bundle L

$l(A, L)$: The dimension of $\Gamma(A, L)$ as a vector space

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