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ON SPAN AND INVERSE LIMITS

By

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1. Introduction.

A compact metric space is called a *compactum* and a connected compactum is called a *continuum*. All maps in this paper are continuous. Let $f: X\rightarrow Y$ be a map between continua. Ingram [2] and Lelek [11] defined the *span, semispan, surjective span,* and *surjective semispan* of f by the following formulas (the map $p_i: X\times X\rightarrow X$ denotes the projection to the i-th factor, $i=1, 2$).

$$\tau=\sigma,\ \sigma_0,\ \sigma^*,\ \sigma_0^*.$$

$$\tau(f)=\sup\left\{c\geqq 0\ \middle|\ \begin{array}{l}\text{there exists a continuum } Z\subset X\times X \text{ such}\\ \text{that } Z \text{ satisfies the condition } \tau) \text{ and}\\ d(f(x), f(y))\geqq c \text{ for each } (x, y)\in Z\end{array}\right\},$$

where the condition τ) is:

$$p_1(Z)=p_2(Z)\ \text{ if } \tau=\sigma, \qquad p_1(Z)\supset p_2(Z)\ \text{ if } \tau=\sigma_0,$$
$$p_1(Z)=p_2(Z)=X\ \text{ if } \tau=\sigma^*, \qquad p_1(Z)=X\ \text{ if } \tau=\sigma_0^*.$$

The span of a continuum X is defined by $\sigma(id_X)$. The other cases are similar. In the same way, we can define the *symmetric span* of f by the formula

$$s(f)=\sup\left\{c\geqq 0\ \middle|\ \begin{array}{l}\text{there exists a continuum } Z\subset X\times X \text{ such that}\\ Z \text{ is symmetric (i.e. } (x, y)\in Z \text{ iff } (y, x)\in Z)\\ \text{and } d(f(x), f(y))\geqq c \text{ for each } (x, y)\in Z\end{array}\right\}.$$

It is a mapping version of symmetric span of a continuum due to J. F. Davis [1].

Let $X=\varprojlim(X_n, p_{n\,n+1})$ be a continuum, where $p_{n\,n+1}: X_{n+1}\rightarrow X_n$. Ingram [2] and [4] showed that $\sigma(X)=0$ if and only if there exists a cofinal subsequence $(n_i)_{i\geqq 1}$ such that $\lim_j \sigma(p_{n_i n_j})=0$ for each $i\geqq 1$. In section 2 of this paper, we will prove a mapping version of this theorem. H. Cook proved essentially that the symmetric span of the dyadic solenoid is zero ([1], p. 134), while its span is positive. The author wishes to thank to the referee for pointing out this fact. In section 3, we generalize this to the poly-adic solenoid. Let f and $g: X\rightarrow Y$ be maps. $d(f, g)$ denotes $\sup\{d(f(x), g(x))\mid x\in X\}$.

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