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APPROXIMATIVE SHAPE IV

—UV^n-MAPS AND THE VIETORIS-SMALE THEOREM—

Dedicated to Professor Yukihiro Kodama on his sixtieth birthday

By

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§ 0. Introduction.

This paper is a continuation of [35-37]. We introduced approximate shape in [35], discussed approximative shape properties of spaces and generalized ANRs in [36], and fixed point theorems in [37]. In this paper we investigate approximative shape properties of maps and show the Vietoris-Smale theorem in shape theory.

Many mathematicians studied UV^n-maps. See the references of Lacher [18] for their studies. Smale [30] gave a Vietoris type theorem for homotopy groups and UV^n-maps, called the Vietoris-Smale theorem. Kozlowski [13] gave a factorization theorem for UV^n-maps. Borsuk introduced approximatively n-connected spaces. This is a basic notion in shape theory. Various Vietoris-Smale theorems in shape theory were given by Bogatyi [2, 3], Dydak [4-7], Kodama [11, 12], Kuperberg [16], Kozlowski-Segal [15] and Morita [27, 28].

In this paper we discuss the following topics: In § 1 we introduce the approximative lifting property and investigate its properties. In § 2 we prove restriction and product theorems for the approximative lifting property. In § 3 we introduce approximatively n-connected maps and give their characterizations. We show the Vietoris-Smale theorem and the Whitehead theorem for approximatively n-connected maps. In § 4 we introduce the approximative extension property. We characterize approximatively n-connected spaces by this property. In § 5 we introduce partial realizations for decomposition spaces. We introduce the approximative full extension property and investigate its properties. In § 6 we show that our approximatively n-connected maps and usual UV^n-maps are equivalent. Hence by using results in § 3 we show the Vietoris-Smale theorem and the Whitehead theorem in shape theory for closed UV^n-maps between paracompacta.

We assume that the reader is familiar with theory of ANRs and shape theory. As reference books we use Hu [10] for theory of ANRs and Mardešić

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