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REMARKS ON ZIL'BER'S RESULTS: ON THE SIMPLICITY OF THE REPRESENTATION OF GROUPS AND RINGS

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§ 0. Introduction.

B. I. Zil'ber made tremendous amount of important works about groups of finite Morley rank. One of the most important works of his is the indecomposability theorem for groups of finte Morley rank, with which he proved that the simplicity of a group of finite Morley rank is preserved by elementary equivalence, and simple groups of finite Morley rank are almost strongly minimal.

We will show with his methods that the simplicity (or irreducibility) of a representation of a connected group or ring (not necessarily associative) over a group of finite Morley rank is preserved by elementary equivalence. As the simplicity of a group or a ring can be considered as the simplicity of its representation on itself, this extends the result of Zil'ber above and also we get the theorem for a ring of finite Morley rank. Berlin and Lascar extended the indecomposability theorem to the superstable case and one of their applications was the preservation theorem for the simplicity of a superstable group ([3]). We can also extend their works to the cases as here with some modification.

We also prove that a simple ring of finite Morley rank is almost strongly minimal. For a simple representation of a group or a ring, we can prove that it is strongly minimal under some stronger assumption and in a different language from above. Our results yield many algebraic examples of the almost strongly minimal structure, hence of the $|L|^+$-categorical structure where L is the language of the structure considering. As for ω_1-categorical ring, Zil'ber made complete classification for associative ring of characteristic zero ([8]), and Rose proved that finite dimensional central algebra over an algebraically closed field is ω_1-categorical ([6]). We can easily find an almost strongly minimal ring which is not associative or central, namely a finite dimensional simple Lie algebra for example.

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