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APPROXIMATIVE SHAPE III
—FIXED POINT THEOREMS—

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§ 0. Introduction.

This paper is a continuation of [**45**-**46**]. We introduced approximative shape in [**45**], and discussed approximative shape properties of spaces and generalized ANRs in [**46**]. In this paper we shall discuss approximative shape properties of maps and fixed point theorems.

The Lefschetz-Hopf fixed point theorem is a well-known fixed point theorem formulated in homological or cohomological terms. It was first discovered by Lefschetz for compact manifolds and then extended by him to manifolds with boundary. Hopf gave a completly different and simple proof for finite polyhedra and then Lefschetz extended it to compact metric ANRs (see Lefschetz [**33**]). It was extended to compact metric AANR_Ms by Granas [**22**], to compact metric AANR_Cs by Clapp [**9**] and to metric AANR_Cs by Powers [**40**].

Borsuk [**3**, **5**] introduced nearly extendable sets, in notation NE-sets, and nearly extendable maps, in notation NE-maps, between compact metric spaces. He [**4**, **6**] showed the Lefschetz-Hopf fixed point theorem for NE-maps and Gauthier [**19**-**21**] extended it to NE-maps between compact spaces.

Borsuk and Ulam [**7**] introduced symmetric products. This notion was generalized as G-product where G is a subgroup of all permutations of coordinates. Maxwell [**36**] showed a fixed point theorem for maps into G-products of finite polyhedra. The Maxwell fixed point theorem contains the Lefschetz-Hopf fixed point theorem as a special case. The Maxwell fixed point theorem is extended to maps into G-products of compact metric ANRs by Masih [**35**] and to maps into G-products of compact metric AANR_Ns by Vora [**42**].

In this paper we investigate the following topics. In § **1** we introduce NE-sets and NE-maps between arbitrary spaces. We show that the notions of ap-approximative movability and NE-sets are equivalent. We show that the notions of AANR_C and NE-sets are equivalent for compact metric spaces, but not for compact spaces. This gives a negative answer to a question of Gauthier [**20**]. In § **2** we show that products, suspensions and cones preserve NE-maps. In § **3**

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