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$\boldsymbol{Z}$-KERNEL GROUPS OF MEASURABLE CARDINALITIES

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$\boldsymbol{Z}$-kernel groups are groups obtained by transfinitely iterating use of direct products and sums starting from the group of integers $\boldsymbol{Z}$. In [2] the author defined type for a $\boldsymbol{Z}$-kernel group and proved its uniqueness for a $\boldsymbol{Z}$-kernel group of cardinality less than the least measurable cardinal.

In the present paper we show the uniqueness of type for more general $\boldsymbol{Z}$-kernel groups, e. g., $\prod_{A_1}\bigoplus_{A_2}\cdots_{A_n}\boldsymbol{Z}$ for arbitrary $A_1, \cdots, A_n$. In addition we show that the $\boldsymbol{Z}$-dual of such a $\boldsymbol{Z}$-kernel group again becomes a $\boldsymbol{Z}$-kernel group. One of our tools is Zimmermann's trick extended for an arbitrary cardinality from group theory and the other is finitely iterated ultrapowers of the universe from set theory. Our notation and terminology are common with [3] and undefined ones are usual ones in group theory [4] and set theory [5].

DEFINITION 1 [1]. A $\boldsymbol{Z}$-kernel group is a group obtained in the following manner:

(1) The group of integers $\boldsymbol{Z}$ is a $\boldsymbol{Z}$-kernel group;

(2) If G_α is a $\boldsymbol{Z}$-kernel gruop for each $\alpha\in A$, then $\prod_{\alpha\in A}G_\alpha$ and $\bigoplus_{\alpha\in A}G_\alpha$ are $\boldsymbol{Z}$-kernel groups, where A is nonempty.

Without loss of generality we may assume that A is an ordinal, since we work in ZFC-set theory.

DEFINITION 2 [2]. A type is a pair $(\mu, P), (\mu, S)$ or (μ, M), where μ is an ordinal. For types (μ, X) and $(\nu, Y), (\mu, X)<(\nu, Y)$ holds if $\mu<\nu$, or $\mu=\nu$ and $X\neq M$ and $Y=M$. We say that μ is the ordinal part of a type (μ, X).

Next we define a proper $\boldsymbol{Z}$-kernel ($\boldsymbol{pZk}$) group with type. We denote type of a $\boldsymbol{pZk}$ group G by typ(G) and the ordinal part of it by typ*(G). A rigorous reader should think that $\boldsymbol{Z}$-kernel groups and $\boldsymbol{pZk}$ groups are not just groups but groups with their definitions. Therefore, when we say that two $\boldsymbol{Z}$-kernel groups are isomorphic, it means that group parts of them are isomorphic.

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