TSUKUBA J. MATH.
Vol. 8 No. 1 (1984). 77—80

M_0-SPACES ARE μ-SPACES

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1. Introduction. The μ-spaces were introduced by K. Nagami [N]. A space X is said to be a μ-space if X is embedded in the countable product of F_σ-metrizable paracompact spaces. The class of M_3-μ-spaces is a harmonious class in dimension theory and is a subclass of hereditary M_1-spaces (see [M] and [T]). Especially every 0-dimensional M_3-μ-space has a σ-closure preserving clopen base. Heath and Junnila [HJ] called such a space an *M_0-space.* Then, what spaces are M_3-spaces to be μ-spaces? There was no result on this question yet. In this paper we shed some light on this question.

Throughout this paper all spaces are assumed to be regular T_1 and all maps are assumed to be continuous. The letter N denotes the positive integers.

2. Results.

THEOREM 2.1. *Let X be an M_3-space with a peripherally compact σ-closure preserving quasi-base. Then X is embedded in the countable product of F_σ-metrizable M_3-spaces and is therefore a μ-space.*

PROOF. Let $\mathcal{B}=\cup\{\mathcal{B}_n : n\in N\}$ be a peripherally compact σ-closure preserving closed quasi-base of X. Let $n\in N$. To construct a space M_n, let us fix n. Let $V(x)=X-\cup\{B\in\mathcal{B}_n : x\notin B\}$ and $\hat{x}=\{y\in X: V(x)=V(y)\}$. Then by [J, Theorem 4.8], there exists a σ-discrete closed refinement $H=\cup\{H_m : m\in N\}$ of $\{\hat{x} : x\in X\}$. By [O, Lemma 3.2], there exist a metrizable space Z and a one-to-one onto map $f: X\to Z$ such that every $f(H_m)$ is a discrete closed family and $f(\mathcal{B}_n)$ is a closure preserving closed family. For $B\in\mathcal{B}_n$, there exists a map $\Psi'_B: f(B)\to I$ such that $\Psi'^{-1}_B(0)=f(\partial B)$, because ∂B is compact, where ∂B denotes the boundary of B. Let $\Psi_B: X\to I$ such that

$$\Psi_B(x)=\Psi'_B\circ f(x) \text{ if } x\in B;\text{ and}$$
$$\Psi_B(x)=0 \text{ if } x\notin B.$$

Received June 11, 1983.
This result was announced in "General Topology Synposium, Japan", December 1983.