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ISOMETRIC IMMERSIONS OF LORENTZ SPACE WITH PARALLEL SECOND FUNDAMENTAL FORMS

By

Martin A. MAGID

Introduction

In a series of papers, [F1], [F2], [F3], [F4], D. Ferus classified submanifolds of euclidean space with parallel second fundamental forms. These submanifolds form an important class of examples. In [F4], Ferus shows that they appear in many topics in differential geometry.

There has been much recent work on parallel submanifolds of other ambient spaces—notably the work of H. Naitoh, [Na1]-[Na4] and M. Takeuchi [T]. An interesting problem is to classify the parallel submanifolds of euclidean spaces equipped with indefinite metrics.

This paper studies special classes of parallel submanifolds in $\boldsymbol{R}_1^m$, Lorentz space of signature $(1, m-1)$ and in $\boldsymbol{R}_2^m$, euclidean space of signature $(2, m-2)$. All umbilical submanifolds are classified, as well as isometric immersions $\boldsymbol{R}^n \to \boldsymbol{R}_1^{n+k}$, $\boldsymbol{R}_1^n \to \boldsymbol{R}_1^{n+2}$ and $\boldsymbol{R}_1^n \to \boldsymbol{R}_2^{n+2}$ with parallel second fundamental forms. These theorems indicate some of the modifications which will be necessary in order to obtain a complete classification.

The preliminary section (0) gives some basic results about indefinite Riemannian geometry. These include an indefinite version of the result of Allendoerfer and Erbacher for reducing the codimension of an isometric immersion with parallel second fundamental form ($\nabla^*\alpha=0$), and an improved version of Petrov's canonical forms for symmetric transformations of Lorentz space.

Section 1 classifies isometric immersions $\boldsymbol{R}_1^2 \to \boldsymbol{R}_1^4$, $\boldsymbol{R}_1^2 \to \boldsymbol{R}_2^4$ and $\boldsymbol{R}^n \to \boldsymbol{R}_1^{n+k}$ with $\nabla^*\alpha=0$. These maps include a flat n-dimensional umbilic with lightlike mean curvature vector (1.4) and the complex circle of radius $\kappa \in \boldsymbol{C}$ (1.12).

Section 2 contains the main classification results. These state that isometric immersions $\boldsymbol{R}_1^n \to \boldsymbol{R}_1^{n+2}$ and $\boldsymbol{R}_1^n \to \boldsymbol{R}_2^{n+2}$ with $\nabla^*\alpha=0$ are either quadratic in nature, like the flat umbilical immersion with lightlike mean curvature vector, or the product of the identity map and previously determined low dimensional maps.

Most of the results in this paper are taken from the author's Brown University

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