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UNIFORM VECTOR BUNDLES OF RANK $(n+1)$ ON P_n

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Introduction.

Here vector bundle (or sometimes bundle) means algebraic vector bundle on an algebraic variety. Every variety is defined over an algebraically closed field K with $ch(K)=0$. We write $\boldsymbol{P}_n:=\boldsymbol{P}_n(K)$. A vector bundle E on $\boldsymbol{P}_n$ is uniform if there exists a sequence of integers $(k; r_1, \cdots, r_k; a_1, \cdots, a_k)$ (called the splitting type of E) with $a_1>\cdots>a_k$ and such that for every line L of $\boldsymbol{P}_n$: $E_L\cong\bigoplus_{i=1}^{K} r_i\mathcal{O}_L(a_i)$. If the rank r of E is low with respect to the dimension n of $\boldsymbol{P}_n$, there are only a few uniform vector bundles of rank r. See [1], [2], [5] for the following

THEOREM. *For $r\leqq n$, $n\geqq 2$, $r=3$ and $n=2$, the uniform vector bundles of rank r on $\boldsymbol{P}_n$ are (up to isomorphism) direct sum of line bundles, $\Omega_{\boldsymbol{P}_n}^1(a)$, $T\boldsymbol{P}_n(b)$, $S^2T\boldsymbol{P}_n(c)$, with a, b, c integers.*

In particular every such bundle is homogeneous, i.e. for every automorphism g of $\boldsymbol{P}_n$, $g^*(E)\cong E$. But for $r\geqq 2n$ there exists uniform vector bundles of rank r on $\boldsymbol{P}_n$ which are not homogeneous. Thus it remains open the range $n+1\leqq r<2n$. Ph. Ellia in [3] proved that a uniform rank-$(n+1)$ vector bundle on $\boldsymbol{P}_n$ is decomposable if $n=3, 4, 5$ or $n=p-1$ where p is a prime number. His methods give also many other partial results on rank-$(n+1)$ vector bundles on $\boldsymbol{P}_n$, giving evidence to the following

TEEOREM 1. *Every uniform vector bundle of rank $n+1$ on $\boldsymbol{P}_n$ is isomorphic either to a direct sum of line bundles or to the direct sum of a line bundle and of $\Omega_{\boldsymbol{P}_n}^1(b)$ or $T\boldsymbol{P}_n(a)$.*

In this paper we prove theorem 1, using the methods of [3]. To pass from [3] to theorem 1 no geometry is involved; the only problems are about roots of unity, roots of polynomials or decomposition of polynomials. Thus the proofs are tricky.

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