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FOLDINGS OF ROOT SYSTEMS AND GABRIEL'S THEOREM

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1. Introduction.

Gabriel's theorem [5] (cf. below for precise statements) was generalized by Dlab-Ringel [3], [4] where Dynkin graphs of type B_n, C_n, F_4, G_2 also enter in the classification together with the graphs of type A_n, D_n, E_n in [5]. We give in this note another generalization of [5] using the fact that B_n, C_n, F_4, G_2 are obtained by the so-called folding-operation from A_n, D_n, E_6. Our formulation is rather similar to the original formulation in [5].

Let Γ be a finite graph. We denote its set of vertices by Γ_0 and its set of edges by Γ_1 (there may be several edges between two vertices and loops joining a vertex to itself). Let Λ be an orientation of Γ. For each $l \in \Gamma_1$ we denote its starting-point by $\alpha(l)$ and its end-point by $\beta(l)$.

For a fixed field k we define a category $\mathcal{L}(\Gamma, \Lambda)$ after Gabriel [5] as follows.

DEFINITION 1. Let (Γ, Λ) be a finite oriented graph. A pair (V, f) is an object of $\mathcal{L}(\Gamma, \Lambda)$ if $V = \{V_\alpha \mid \alpha \in \Gamma_0\}$ is a family of finite-dimensional vector spaces over k, and $f = \{f_l : V_{\alpha(l)} \to V_{\beta(l)} \mid l \in \Gamma_1\}$ is a family of k-linear mappings. $(V, f) \xrightarrow{\varphi} (W, g)$ is a morphism if $\varphi = \{\varphi_\alpha : V_\alpha \to W_\alpha \mid \alpha \in \Gamma_0\}$ is a family of k-linear mappings such that for each $l \in \Gamma_1$ the following diagram

$$\begin{tikzcd} V_{\alpha(l)} \arrow[r, "f_l"] \arrow[d, "\varphi_{\alpha(l)}"'] & V_{\beta(l)} \arrow[d, "\varphi_{\beta(l)}"] \\ W_{\alpha(l)} \arrow[r, "g_l"'] & W_{\beta(l)} \end{tikzcd}$$

commutes.

The category $\mathcal{L}(\Gamma, \Lambda)$ is naturally an abelian category and in this category the theorem of Krull-Remak-Schmidt about the essential uniqueness of direct-

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