TSUKUBA J. MATH.
Vol. 3, No. 1 (1979), 123–128

REMARK ON LOCALIZATIONS OF NOETHERIAN RINGS WITH KRULL DIMENSION ONE

By

Hideo SATO

Let R be a left noetherian ring with left Krull dimension α. For a left R-module M which has Krull dimension, we denote its Krull dimension by $K\text{-}dim\, M$ in this note. In the previous paper [6], we have shown that the family $\boldsymbol{F}_{\beta}(R)=\{{}_{R}I\subseteq R \mid K\text{-}dim\, R/I\leq\beta\}$ is a left (Gabriel) topology on R for any ordinal $\beta<\alpha$. We are most interested in the case when R is (left and right) noetherian, $\alpha=1$ and $\beta=0$. Let R be such a ring and we denote $\boldsymbol{F}_{0}(R)$ by $\boldsymbol{F}$. Let A be the artinian radical of R. Then Lenagan [3] showed that R/A has a two-sided artinian, two-sided classical quotient ring $Q(R/A)$. In this note, we shall show that $R_{\boldsymbol{F}}$, the quotient ring of R with respect to $\boldsymbol{F}$, is isomorphic to $Q(R/A)$ as ring and we shall investigate a more precise structure of $R_{\boldsymbol{F}}$ under some additional assumptions.

In this note, a family of left ideals of R is said to be a topology if it is a Gabriel topology in the sense of Stenström's book [7]. So a perfect topology in this note is corresponding to a perfect Gabriel topology in [7]. Let $\boldsymbol{G}$ be a left topology on R, and M a left R-module. A chain of submodules of M;

$$M_{0}\supseteq M_{1}\supseteq\cdots\cdots\supseteq M_{r}$$

is called a $\boldsymbol{G}$-chain if each M_{i-1}/M_{i} is not a $\boldsymbol{G}$-torsion module. A $\boldsymbol{G}$-chain of M is said to be maximal if it has no proper refinement of $\boldsymbol{G}$-chain.

The following lemma can be proved easily.

LEMMA 1. *If ${}_{R}M$ has a finite maximal $\boldsymbol{G}$-chain of length r, then any $\boldsymbol{G}$-chain of M has a finite length s and $s\leq r$.*

Hence we can give a definition of $\boldsymbol{G}$-dimension of M, denoted by $\boldsymbol{G}\text{-}dim\, M$, as follows; if M has a finite maximal $\boldsymbol{G}$-chain of length r, define $\boldsymbol{G}\text{-}dim\, M=r$, and $\boldsymbol{G}\text{-}dim\, M=\infty$ otherwise.

COROLLARY 2. *For any short exact sequence of R-modules;*

$$0\rightarrow M'\rightarrow M\rightarrow M''\rightarrow 0$$

we have $\boldsymbol{G}\text{-}dim\, M=\boldsymbol{G}\text{-}dim\, M'+\boldsymbol{G}\text{-}dim\, M''$.

Received October 27, 1978. Revised February 6, 1979