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ON THE BAHADUR REPRESENTATION OF SAMPLE QUANTILES FOR MIXING PROCESSES

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1. Introduction.

The asymptotic almost sure (a.s.) representation of sample quantiles for independent and identically distributed random variables was firstly established by Bahadur [1]. Kiefer [5, 6] obtained further developments on this line and also investigated the a.s. representation of quantile process. Here we remark that the representation of sample quantiles in the sense of in probability was obtained first of all by Okamoto [7]. The extensions of Bahadur's by relaxing the assumption of independence of the basic random variables have been studied by a number of authors. Especially, Sen [10] obtained completely analogous results to Bahadur's one for stationary ϕ-mixing processes. The object of the present paper is to show that the Bahadur representation holds, but with a slightly different order of the remainder term, for stationary sequences of strong mixing random variables. We also consider the Bahadur representation for absolutely regular processes and the a.s. representation of quantile processes for ϕ-mixing and strong mixing processes.

Let $\{X_n, n\geqq 1\}$ be a strictly stationary sequence of random variables defined on a probability space $(\Omega, \mathscr{B}, P)$. We shall say that the sequence $\{X_n\}$ is ϕ-mixing if

(1.1) $$\sup|P(AB)-P(A)P(B)|/P(A)=\phi(n)\downarrow 0 \quad (n\to\infty),$$

absolutely regular if

(1.2) $$E\{\sup|P(B|\mathscr{M}_1^k)-P(B)|\}=\beta(n)\downarrow 0 \quad (n\to\infty),$$

and strong mixing if

(1.3) $$\sup|P(AB)-P(A)P(B)|=\alpha(n)\downarrow 0 \quad (n\to\infty).$$

Here the supremum is taken over all $A\in\mathscr{M}_1^k$ and $B\in\mathscr{M}_{k+n}^\infty$, and $\mathscr{M}_a^b$ denotes the σ-field generated by $X_n(a\leqq n\leqq b)$. Among these conditions (1.1)-(1.3), the following inequalities hold:

$$\alpha(n)\leqq\beta(n)\leqq\phi(n).$$

For a discussion of mixing conditions, see for example, Ibragimov and Linnik [3]. In late sections, in addition to (1.3), we may need the following condition:

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