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A NOTE ON A FORMALIZED ARITHMETIC WITH FUNCTION SYMBOLS ′ AND +.

By

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Introduction.

Let $\mathfrak{L}_0$ be the first order language with function symbols ′, + and the equality symbol =. By $\mathfrak{L}$ we denote the first order language obtained from $\mathfrak{L}_0$ by adding a ternary predicate symbol P. The theory in $\mathfrak{L}$ with the following axioms and axiom schemata is signified by $\mathfrak{N}$.

(N- 1) $\forall x \neg(x'=0)$.

(N- 2) $\forall x \forall y(x'=y' \supset x=y)$.

(N- 3) $\forall x(x+0=x)$.

(N- 4) $\forall y \forall y(x+y'=(x+y)')$.

(N- 5) $\forall x P(x, 0, 0)$.

(N- 6) $\forall x \forall y \forall z\{P(x, y, z) \supset P(x, y', z+x)\}$.

(N- 7) $\forall x \forall y \forall z \forall w\{(P(x, y, z) \land P(x, y, w)) \supset z=w\}$.

(N- 8) $\forall x(x=x)$.

(N- 9) $\forall x \forall y\{x=y \supset (\mathfrak{A}(x) \supset \mathfrak{A}(y))\}$.

(N-10) $\{\mathfrak{A}(0) \land \forall x((\mathfrak{A}(x) \supset \mathfrak{A}(x')))\} \supset \forall x \mathfrak{A}(x)$.

(N-11) $s=t$, where $s=t$ is valid.

For a term t, $\mathrm{b}(t)$ means the number of occurrences of bound varibles in t. For a formula $\mathfrak{A}$, $\mathrm{b}(\mathfrak{A})$ is defined inductively as follows. 1. $\mathrm{b}(r=s)=\max(\mathrm{b}(r), \mathrm{b}(s))$. 2. $\mathrm{b}(P(r, s, t))=\max(\mathrm{b}(r), \mathrm{b}(s), \mathrm{b}(t))$. 3. $\mathrm{b}(\neg\mathfrak{A})=\mathrm{b}(\mathfrak{A})$. 4. $\mathrm{b}(\mathfrak{A}\land\mathfrak{B})=\mathrm{b}(\mathfrak{A}\lor\mathfrak{B})=\max(\mathrm{b}(\mathfrak{A}), \mathrm{b}(\mathfrak{B}))$. 5. $\mathrm{b}(\forall x\mathfrak{A})=\mathrm{b}(\exists x\mathfrak{A})=\mathrm{b}(\mathfrak{A})$.

In [3] we proved that:

For any formula $\mathfrak{A}(a)$ *of* $\mathfrak{L}$; *if there is a number m such that, for any natural number n, there exists a proof* $\mathfrak{P}$ *of* $\mathfrak{A}(\bar{n})$ *in* $\mathfrak{N}$ *with the following properties (1) and (2), then* $\forall x\mathfrak{A}(x)$ *is provable in* $\mathfrak{N}$.

(1) *The length of* $\mathfrak{P}$ *is less than m.*

(2) *For any induction schema* $\mathfrak{B}$ *in* $\mathfrak{P}$ *which is not a formula of* $\mathfrak{L}_0$, $\mathrm{b}(\mathfrak{B}) \leq m$.

The purpose of this paper is to prove the following theorem.

THEOREM. *There are a formula* $\mathfrak{A}(a)$ *and a natural number M such that*: (a)

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