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A REMARK ON NONNEGATIVELY CURVED HOMOGENEOUS KÄHLER MANIFOLDS

By

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Introduction.

The aim of this paper is to show the following

THEOREM *If a compact connected homogeneous Kähler manifold* (M, g) *is of nonnegative curvature, then it is a Kählerian direct product of a flat complex torus* (T, g_0) *and a Hermitian symmetric space of compact type* (M', g'); $(M, g)\cong(T, g_0)\times(M', g')$.

Here, a Kähler manifold (M, g) is called homogeneous if the isometry group $I(M, g)$ acts on M transitively. And a Kähler manifold is said to be of nonnegative curvature when the sectional curvature K_σ is nonnegative for any plane section σ.

Hermitian symmetric spaces of compact type give typical examples for compact Kähler manifolds of nonnegative curvature (Helgason [2]).

For a Kähler manifold (M, g), the holomorphic bisectional curvature $H_{\sigma,\tau}$ for holomorphic plane sections σ and τ is defined by

$$H_{\sigma,\tau}=g(R(X, IX)IY, Y),\ \sigma=X\wedge IX,\ \tau=Y\wedge IY,\ g(X, X)=g(Y, Y)=1,$$

where R is the curvature tensor and I is the complex structure (Kobayashi and Nomizu [5]). Since $g(R(X, IX)IY, Y)=g(R(X, Y)Y, X)+g(R(X, IY)IY, X)$, the holomorphic bisectional curvature $H_{\sigma,\tau}$ is written by a sum of two sectional curvatures up to nonnegative constant factors. Thus, if a Kähler manifold is of nonnegative curvature, then the holomorphic bisectional curvature $H_{\sigma,\tau}$ is also nonnegative for any holomorphic plane sections σ and τ.

From a theorem of Matsushima [6], a compact connected homogeneous Kähler manifold (M, g) is a Kählerian direct product of a flat complex torus (T, g_0) and a Kähler C-space (M', g'); $(M, g)\cong(T, g_0)\times(M', g')$, where a Kähler C-space is by definition a compact simply connected homogenous Kähler manifold.

If the compact connected homogeneous Kähler manifold (M, g) is of nonnegative curvature, so is the Kähler C-space (M', g'). Therefore, Theorem is a direct conclusion of the following proposition.

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