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FUNDAMENTAL SOLUTION OF CAUCHY PROBLEM FOR HYPERBOLIC SYSTEMS AND GEVREY CLASS

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§ 1. Introduction

We consider a first order partial differential operator $L_{t,x}=\frac{\partial}{\partial t}+\sum_{j=1}^{n}A_j(t,x)\frac{\partial}{\partial x_j}+B(t,x)$ in $\Omega=[0,T]\times R^n$, whose coefficients are $m\times m$-matrices. We call a fundamental solution corresponding to the operator $L_{t,x}$, a distribution satisfying the following, $\tau\in[0,T)$, fixed,

$$(1.1)\qquad \begin{cases} L_{t,x}K(t,x.\tau,y)=0, & t\in(0,T) \\ K(\tau,x,\tau,y)=\delta(x-y)I, \end{cases}$$

here $\delta(x)$ denotes the n-dimentional Dirac distribution and I the indentity matrix. We require that the multiplicity of each characteristic remains constant in a region $\Omega=[0,T]\times R^n$ and that the characteristic matrix $A(t,x,\xi)=\Sigma A_j(t,x)\xi_j$ is diagonalizable for (t,x) in Ω and ξ in $R^n\setminus 0$. Moreover we suppose that the coefficients $A_j(t,x)$ and $B(t,x)$ are in Gevrey class $\gamma_s(\Omega)(s\geq 1)$.

Our aim is to construct globally in Ω a fundamental solution for the operator $L_{t,x}$ of this type. When T is small, Lax [12] treated this problem. In the case of analytic coefficients, Leray [13] and Mizohata [19] analyzed locally a fundamental solution of hyperbolic systems. When T is large, Ludwig [15] extended the interval of existence for a fundamental solution by use of it's semi-group property. We shall give a more precise expression of a fundamental solution than these of Ludwig. It should be remarked that Duistermaat [3] has recently constructed globally a fundamental solution of the Cauchy problem, applying the theory of Fourier integral operators of Hörmander and Duistermaat [4], [9].

In the first step we shall construct asymptotically a fundamental solution and in the second step we shall obtain successive estimates of it's expansion by use of the method of Mizohata [18], [19] and Hamada [7], [8]. We shall determine the wave front set in Gevrey class of a fundamental solution following the definition of Hörmander [10].

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