TSUKUBA J. MATH.
Vol. 1 (1977), 137–148

ON THE NILPOTENCY INDICES OF THE RADICALS OF GROUP ALGEBRAS OF *P*-GROUPS WHICH HAVE CYCLIC SUBGROUPS OF INDEX *P*

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Let K be a field with characteristic $p>0$, G a finite group, KG the group algebra of G over K and $J(KG)$ the radical of KG. We are interested in relations between ring-theoretical properties of KG and the structure of G. Particularly, in the present paper we shall study the nilpotency index $t(G)$ of $J(KG)$, which is the least positive integer $t(G)$ such that $J(KG)^{t(G)}=0$.

For a finite p-group P of order p^{r}, S. A. Jennings [3] showed that $r(p-1)+1\leqq t(P)\leqq p^{r}$. Recently K. Motose and Y. Ninomiya [7] determined all p-groups P of order p^{r} such that $t(P)$ are the lower bound $r(p-1)+1$ or the upper bound p^{r}. In fact they proved that for a p-group P of order p^{r} with $r\geqq 1$, $t(P)=r(p-1)+1$ if and only if P is elementary abelian and that $t(P)=p^{r}$ if and only if P is cyclic. So in this paper we shall investigate p-groups P of order p^{r} such that $t(P)$ are not necessarily equal to the lower bound $r(p-1)+1$ or the upper bound p^{r}. By the results of K. Motose [6, Theorem], K. Motose and Y. Ninomiya [7, Theorem 1] it follows that when P is an abelian p-group of order p^{r} with $r\geqq 2$, the secondarily highest nilpotency index $t(P)$ of $J(KP)$ is $p^{r-1}+p-1$ and in this case P is not cyclic and has a cyclic subgroup of index p. Our main result of §1 is a generalization of the above fact. This can be stated as follows: For an arbitrary p-group P of order p^{r} with $r\geqq 2$, the next conditions are equivalent;

(i) $t(P)=p^{r-1}+p-1$.

(ii) $p^{r-1}<t(P)<p^{r}$.

(iii) P is not cyclic and has a cyclic subgroup of index p.

There is a problem that when the value of $t(G)$ is given, what type is G? About this there are some solutions ([9], [7]). D. A. R. Wallace [9] determined all finite groups G with the property $t(G)=2$. Further, K. Motose and Y. Ninomiya [7] determined all finite p-solvable groups G such that $t(G)=3$. In connection with this in §2 we shall have all p-groups P such that $t(P)=4, 5$ or 6 by calculating

Received April 11, 1977. Revised October 19, 1977