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UNIVERSAL CENTRAL EXTENSIONS OF LINEAR GROUPS OVER RINGS OF NON-COMMUTATIVE LAURENT POLYNOMIALS, ASSOCIATED K_1-GROUPS AND K_2-GROUPS

By

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1. Introduction

Many researchers have studied the structure of general linear groups and their elementary subgroups over fields F or commutative rings R. They also have analyzed associated lower K-groups, for example [4] and [9]. Needless to say, general linear groups are important objects and have many applications in various areas of mathematics, but they particularly have much to do with Lie theory; Lie groups, Lie algebras and their representations. Lower K-groups also play an important role as a certain invariant.

In this paper, we treat some rings D_τ of non-commutative Laurent polynomials over division rings D (cf. Section 2). Here τ is an automorphism of D. We note that the ring we use generalizes the one which is studied in [4] and [9]. When $D = F$ and $\tau = id$, our discussion is just a subject of loop groups which are applied in the theory of affine Kac-Moody Lie algebras, and this is surveyed in [5] for example. On the other hand, the corresponding linear group was studied in the case when D is the field of formal power series and τ is not trivial (cf. [6], [8]), which is deeply related to the theory of extended affine Lie algebras (cf. [1], [10], [11]).

Our main object in this study is the following exact sequence.

$$1 \to K_2(n, D_\tau) \to St(n, D_\tau) \xrightarrow{\phi} GL(n, D_\tau) \to K_1(n, D_\tau) \to 1.$$

We reveal the structure of groups in the above sequence. We first describe an existence of a Tits system in the elementary subgroup $E(n, D_\tau)$ of the general linear group $GL(n, D_\tau)$ and the associated Steinberg group $St(n, D_\tau)$ in Section 2

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