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A MATSUMOTO-TYPE THEOREM FOR LINEAR GROUPS OVER SOME COMPLETED QUANTUM TORI

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0. Introduction

Let $K = F((X_1))$ be a field of formal power series in variable X_1 over an arbitrary field F. We fix an element $q \in F^\times$, and let $K_q = K[X_2, X_2^{-1}]$ be the ring of Laurent polynomials in variable X_2 over K with the relation $X_2X_1 = qX_1X_2$ (cf. Section 1). We call this K_q the completed quantum torus associated with $q \in F^\times$. For $\dot{\ell} \in \mathbf{Z}_{\geq 2}$, we let $A_{\dot{\ell}-1}$ be a Cartan matrix with simple roots $\Pi = \{\alpha_1, \dots, \alpha_{\dot{\ell}-1}\}$, and let $A_{\dot{\ell}-1}^{(1)}$ be an affine Cartan matrix of tier number 1 with affine simple roots $\Pi_1 = \{a_1 = (\alpha_1, 0), \dots, a_{\dot{\ell}-1} = (\alpha_{\dot{\ell}-1}, 0), a_0 = (-\alpha_0, 1)\}$, where α_0 is the highest root of the root system of type $A_{\dot{\ell}-1}$ with respect to Π. Let $M(\dot{\ell}, K_q)$ be the ring of $\dot{\ell} \times \dot{\ell}$ matrices with entries in K_q, and we let $GL(\dot{\ell}, K_q)$ be the multiplicative group of $M(\dot{\ell}, K_q)$. Then we can construct the elementary subgroup $E(A_{\dot{\ell}-1}^{(1)}, K)_q$ of $GL(\dot{\ell}, K_q)$, and the affine Steinberg group $St(A_{\dot{\ell}-1}^{(1)}, K)_q$ associated with $q \in F^\times$. Let $K_2(A_{\dot{\ell}-1}^{(1)}, K)_q$ be the kernel of the canonical homomorphism of $St(A_{\dot{\ell}-1}^{(1)}, K)_q$ onto $E(A_{\dot{\ell}-1}^{(1)}, K)_q$, and we have the fact that $K_2(A_{\dot{\ell}-1}^{(1)}, K)_q$ is central (cf. [17]). Using these notations, we obtain the main result below:

THEOREM. *$K_2(A_{\dot{\ell}-1}^{(1)}, K)_q$ is isomorphic to the abelian group L generated by the symbols $c_a(u,v)$ and $d(w)$ for all $a \in \Pi_1$, $u, v \in K^\times$ and $w \in K_{q,X_2}^\times = \langle u \in K^\times \mid X_2uX_2^{-1} = u \rangle$ with the following defining relations*:

(L1) $c_a(u,v)c_a(uv,t) = c_a(u,vt)c_a(v,t)$

(L2) $c_a(1,1) = 1$

(L3) $c_a(u,v) = c_a(u^{-1}, v^{-1})$

(L4) $c_a(u,v) = c_a(u,(1-u)v)$ with $u \neq 1$

(L5) $c_a(u, v(ab)) = c_b(u(ba), v)$

(L6) $c_{ab}(u,v)$ *is bimultiplicative*

(LD) $d(w)d(x) = d(wx)c_{a_1}(w,x)c_{a_0}(x,w) = d(wx)c_{a_1}(x,w)c_{a_0}(w,x)$

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