TSUKUBA J. MATH.
Vol. 30 No. 2 (2006), 383–400

THE INTEGRATED DENSITY OF STATES OF ONE-DIMENSIONAL RANDOM SCHRÖDINGER OPERATOR WITH WHITE NOISE POTENTIAL AND BACKGROUND

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1. Introduction

We consider the Integrated Density of States (IDS), $N(\lambda)$, $\lambda \in \mathbf{R}$, of the fomally defined operator H,

$$H = -\frac{1}{r(t)}\frac{d}{dt}\left(\frac{1}{p(t)}\frac{d}{dt}\right) + \frac{q(t)}{r(t)} + \frac{cB'(t)}{r(t)}, \quad 0 \le t < \infty, \tag{1.1}$$

i.e., the limit of the normalized distribution function of the eigenvalues of H_l which is the restriction of H to $L^2((0,l) : r(t)\,dt)$ under the boundary conditions,

$$(b.c)_{\alpha,\beta} \quad \begin{cases} \varphi(0)\cos\alpha - \frac{1}{p(0)}\varphi'(0)\sin\alpha = 0, \\ \varphi(l)\cos\beta - \frac{1}{p(l)}\varphi'(l)\sin\beta = 0, \end{cases}$$

where $(B(t))_{t\ge 0}$ is the standard Brownian motion and $B'(t)$ is the derivative of its sample function, namely the white noise. $(p(t))_{t\ge 0}$, $(q(t))_{t\ge 0}$ and $(r(t))_{t\ge 0}$ are bounded semi-martingales which we shall call the background, and c is a *coupling constant*.

$N(\lambda)$ is defined by

$$N(\lambda) := \lim_{l\to\infty} \frac{1}{l} N(l,\lambda,\omega),$$

where we denote by $N(l,\lambda,\omega)$ the number of eigenvalues of H_l which are less than or equal to λ.

The main purpose of this paper is to improve Theorem of [5] and Theorem (b) of [12] cited below, simplifying their proofs at the same time.

PROPOSITION 1.1 ([5]). *Suppose that* $p(t) \equiv 1$, $q(t) \equiv 0$, $r(t) \equiv 1$ *and* $c = 1$. *Then*

Received May 2, 2005.
Revised March 23, 2006.