TSUKUBA J. MATH.
Vol. 30 No. 2 (2006), 259–271

SMOOTHLY SYMMETRIZABLE COMPLEX SYSTEMS AND THE REAL REDUCED DIMENSION

By

Tatsuo Nishitani and Jean Vaillant

1 Introduction

Let L be a first order system

$$L(x, D) = \sum_{j=1}^{n} A_j(x) D_j$$

where $A_1 = I$ is the identity matrix of order m and $A_j(x)$ are $m \times m$ complex valued smooth matrix functions. In this note we continue to study the question when we can smoothly symmetrize $L(x, D)$. In particular we discuss about the question whether we can smoothly reduce $L(x, D)$ to a hermitian system if $L(x, D)$, at every frozen x, is similar to hermitian system as a constant coefficient system. In [2], [3] the same question for real systems was studied. Let $L(x, \xi)$ be the symbol of $L(x, D)$: Let us denote

$$L(x, \xi) = (\theta_j^i(x, \xi))$$

which is a $m \times m$ complex valued matrix. We set

$$d(L(x, \cdot)) = \dim \mathrm{span}_{\mathbf{R}}\{\mathrm{Re}\, \theta_j^i(x, \cdot), \mathrm{Im}\, \theta_j^i(x, \cdot)\}$$

which is called the real reduced dimension of L at x.

Our aim in this note is to prove

Theorem 1.1. *Let $m \geq 2$. Assume that at every x near $\bar{x}$ there exists $S(x)$ which is possibly non smooth in x such that $S(x)^{-1} L(x, \xi) S(x)$ is hermitian for every ξ and the real reduced dimension of $L(\bar{x}, \cdot) \geq m^2 - m + 2$. Then there is a smooth $T(x)$ defined near $\bar{x}$ such that*

$$T(x)^{-1} L(x, \xi) T(x)$$

is hermitian for any ξ and for any x near $\bar{x}$.

Received October 14, 2004.