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ENERGY METHOD FOR NUMERICAL ANALYSIS

By

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Introduction

"Energy Inequality" played an essential role in the study of partial differential equations throughout 20-th century. Especially, it is a reliable paradigm that existence of solutions of a problem comes from energy inequality on its adjoint problem.

Recently, we found that the energy method to prove existence of solutions involves the numerical method. In other words, we can say that numerical approximation of solutions comes from energy inequalities on adjoint problems ([1], [2]). We will state its summary in Chapter 1. In Chapter 2, we consider nonlinear problems, where Sobolev's imbedding theorem in general type plays an essential role ([3], [4]). Our proof of existence of solutions suggests a method of numerical approximation of solutions.

Chapter 1. Numerical Approximation in Linear Case

Let us consider a linear boundary value problem as follows.

PROBLEM: To seek a solution $u \in L^2(\Omega)$ satisfying

$$\text{(P)} \qquad \begin{cases} Au = f & \text{in } \Omega \\ B_j u = f_j & \text{on } \Gamma \ \ (j \in J), \end{cases}$$

for given data $\{f, f_j \ (j \in J)\}$, where

(i) A is a linear partial differential operator of order m with smooth coefficients,

(ii) B_j is a linear partial differential operator of order j $(j \in J)$ with smooth coefficients, $(J \subset \{0, 1, \dots, m-1\})$,

(iii) Ω is a bounded domain in $\mathbf{R}^n$ with smooth boundary Γ,

(iv) Γ is non-characteristic for $\{A, B_j \ (j \in J)\}$.

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