TSUKUBA J. MATH.
Vol. 28 No. 2 (2004), 437–453

RESTRICTED ENERGY INEQUALITIES AND NUMERICAL APPROXIMATIONS

By

Reiko SAKAMOTO

Introduction

Let $\{A, B_j\}$ be linear partial differential operators. Let $\Omega(\subset \boldsymbol{R}^n)$ be a bounded domain with smooth boundary Γ. Our boundary value problem is to find $u \in L^2(\Omega)$ satisfying

$$(P) \qquad \begin{cases} Au = f & \text{in } \Omega, \\ B_j u = f_j & \text{on } \Gamma \ (j \in J) \end{cases}$$

for given data $\{f, f_j\}$. We are particularly interested in a method of numerical approximation of solutions of (P).

The problem (P) is closely connected with its adjoint problem (P^*). The adjoint problem is to find $v \in L^2(\Omega)$ satisfying

$$(P^*) \qquad \begin{cases} A^* v = g & \text{in } \Omega \\ \mathscr{B}_j^* v = g_j & \text{on } \Gamma \ (j \in J^*) \end{cases}$$

for given data $\{g, g_j\}$.

Recently, it has become clear that a solution $u \in L^2(\Omega)$ of (P) can be constructed numerically, assuming an energy inequality

$$(E^*) \qquad \|v\| \leqq C\left(\|A^* v\| + \sum_{j \in J^*} \langle \mathscr{B}_j^* v|_\Gamma \rangle_{\mu_j}\right) \quad (v \in H^q(\Omega))$$

([1]).

Here we have two questions:

(1) In case when L^2-solutions of (P) are not unique, how can we characterize the solution, obtained in [1]?

(2) In case when L^2-solutions of (P^*) are not unique, (E^*) can not be satisfied. Is there any numerical method to approach to one of solutions of (P)?

Received May 6, 2003.
Revised October 20, 2003.