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QUANTIFIER ELIMINATION RESULTS FOR PRODUCTS OF ORDERED ABELIAN GROUPS

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1 Introduction

Komori [1] introduced the notion of semi-discrete ordered Abelian group with divisible infinitesimals. Roughly speaking, such groups are products of a $\boldsymbol{Z}$-like group and a $\boldsymbol{Q}$-like group. In [1], he showed that such groups are axiomatized by his set SC of axioms. In fact he showed that SC is complete and admits quantifier elimination (QE) in some language expanding $L_{\mathrm{og}} = \{0, +, -, <\}$. In this paper, we shall evolve his study and prove QE for products of ordered Abelian groups H and K, where H admits QE and K is divisible. However, like him, we need to expand the language slightly. First let us explain Komori's axiom. SC is the following set of sentences:

1. the axioms for ordered Abelian groups;
2. the axioms for a semi-discrete ordering
$$0 < 1, \quad \forall x(2x < 1 \vee 1 < 2x);$$
3. the axioms for infinitesimals
$$\forall x(2x < 1 \rightarrow nx < 1) \quad (n = 2, 3, \ldots);$$
4. the axioms for D_n's
$$\forall x(D_n(x) \leftrightarrow \exists y \exists z(-1 < 2z < 1 \wedge x = ny + z) \quad (n = 2, 3, \ldots)$$
$$\forall x(D_n(x) \vee D_n(x+1) \vee \cdots \vee D_n(x+n-1)) \quad (n = 2, 3, \ldots);$$
5. the axioms for divisible infinitesimals
$$\forall x(-1 < 2x < 1 \rightarrow \exists y(x = ny) \quad (n = 2, 3, \ldots);$$
6. the axiom for existence of infinitesimals
$$\exists x(0 < x < 1);$$

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