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DIRICHLET-NEUMANN PROBLEM IN A DOMAIN WITH PIECEWISE-SMOOTH BOUNDARY

By

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Introduction

The Dirichlet boundary value problem has been considered for various types of partial differential operators in a domain Ω with various types of non-smooth boundaries ([1], [2], etc.).

In this paper, we assume that Ω is a smooth ρ-manifold in $\boldsymbol{R}^n$, defined in § 1, whose boundary is divided into a finite number of smooth surfaces:

$$\partial\Omega = \bigcup_{i=1}^{h} \bar{\Gamma}_i = \left(\bigcup_{i\in D} \bar{\Gamma}_i\right) \cup \left(\bigcup_{i\in N} \bar{\Gamma}_i\right) \quad (D \cap N = \phi).$$

In § 2, we consider an elliptic partial differential equation of 2-nd order in Ω with Dirichlet boundary conditions on Γ_i $(i \in D)$ and Neumann boundary conditions on Γ_i $(i \in N)$:

$$(\mathrm{P}) \quad \begin{cases} Au = f & \text{in } \Omega, \\ u = g^{(i)} & \text{on } \Gamma_i \ (i \in D), \\ B_i u = h^{(i)} & \text{on } \Gamma_i \ (i \in N), \end{cases}$$

where $\{B_i\}$ are differential operators of 1-st order. We consider weak solutions, i.e. $\mathscr{H}$-weak solutions in the sense of [3]. Existence of $\mathscr{H}$-weak solutions depends on weak energy estimates for adjoint problems. Therefore our aim in this paper is to obtain weak energy estimates for adjoint problems. Weak energy estimates for (P) means

$$\|u\|_{L^2(\Omega)} \leqq C\left\{\|Au\|_{L^2(\Omega)} + \sum_{i\in D} \|u\|_{H^{\sigma-1}(\Gamma_i)} + \sum_{i\in N} \|B_i u\|_{H^{\sigma-2}(\Gamma_i)}\right\} \quad (\forall u \in H^\sigma(\Omega))$$

for some large integer σ.

In § 3, we consider a hyperbolic partial differential equation of 2-nd order in $(0, T) \times \Omega$ with Dirichlet boundary conditions on $(0, T) \times \Gamma_i$ $(i \in D)$, Neumann boundary conditions on $(0, T) \times \Gamma_i$ $(i \in N)$ and initial conditions on $\{t = 0\} \times \Omega$.

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