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ON STRONGLY ALMOST HEREDITARY RINGS

By

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M. Harada defined an almost projective module in [8] and showed that semisimple rings, serial rings, QF-rings and H-rings are well-characterized by the property of an almost projective module in [8], [9]. Using an almost projective module he further considered the following generalized condition of a hereditary ring in [7]:

$(*)_r$ Every submodule of a finitely generated projective right R-module is almost projective.

In this paper we call an artinian ring R a *right strongly almost hereditary ring* (abbreviated *right SAH ring*) if R satisfies $(*)_r$. On the other hand, an artinian hereditary ring is characterized by the following equivalent conditions:

(a) Every submodule of a projective right R-module is also projective;

(b) every submodule of a projective left R-module is also projective;

(c) every factor module of an injective right R-module is also injective;

(d) every factor module of an injective left R-module is also injective.

In section 2 we consider the following generalized condition of (c):

$(*^\sharp)_r$ Every factor module of an injective right R-module is a direct sum of an injective module and finitely generated almost injective modules.

Similarly we define $(*^\sharp)_l$ for left R-modules. The first aim of this paper is to show that an artinian ring R is right SAH if and only if R satisfies $(*^\sharp)_l$. But we see that the equivalence between a right SAH ring and an artinian ring which satisfies $(*^\sharp)_r$ does not hold in general.

In [7] M. Harada further considered the following two stronger conditions than $(*)_r$:

$(**)_r$ The Jacobson radical of M is almost projective for any finitely generated almost projective right R-module M;

$(***)_r$ every submodule of a finitely generated almost projective right R-module is also almost projective.

And he showed that an artinian ring R satisfies $(**)_r$ iff it satisfies $(***)_r$. In section 3 we consider the following generalized conditions of (c):

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