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A NOTE ON THE WAVE PACKET TRANSFORMS

Dedicated to Professor Kunihiko Kajitani on the occasion of his sixtieth birthday

By

Takashi ŌKAJI

1. Introduction

The notion of wave front sets of distributions is important in the theory of partial differential equations since it was introduced by M. Sato and L. Hörmander in both the analytic category and the C^∞ one. The FBI transformation, introduced by Bros-Iagolnitzer and Sjöstrand, is very useful to give their characterization in these two categories.

On the other hand, G. B. Folland ([2]) introduced the notion of wave packet transforms $\mathscr{P}_\phi$ corresponding to each member ϕ of the Schwartz space $\mathscr{S}(\boldsymbol{R}^n)$. In this context the standard FBI transformation can be viewed as the wave packet transform corresponding to the Gaussian function $e^{-|x|^2/2}$. In his book he developed a certain symbol calculus related to ϕ when ϕ is an arbitrary nontrivial even function. Furthermore, as its application, he proved that if ϕ is an arbitrary nontrivial even function, wave front sets can be characterized by the wave packet transforms. He raised there an open question whether it is necessary to assume that ϕ is even or not for the symbol calculus.

Our aim is to present a different type of sufficient condition on ϕ. This condition means that ϕ is not necessarily even. Furthermore, we also discuss the H^s wave front sets in terms of wave packet transforms.

2. Wave Packet Transforms

First of all, we recall the definition of the wave front set of distributions, which can give a precise description of the local smoothness properties of distributions. Let $\Omega \subset \boldsymbol{R}^n$ be open and $u \in \mathscr{D}'(\Omega)$. Consider a couple $(x_0, \xi_0) \in \Omega \times \dot{\boldsymbol{R}}^n$. Here and

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