TSUKUBA J. MATH.
Vol. 25 No. 1 (2001), 165–177

# SMOOTHLY SYMMETRIZABLE SYSTEMS AND THE REDUCED DIMENSIONS

Dedicated to Professor Kunihiko KAJITANI on his sixtieth birthday

By

Tatsuo NISHITANI and Jean VAILLANT

## 1. Introduction

In this note we study a first order system

$$L(x, D) = \sum_{j=1}^{n} A_j(x) D_j$$

where $A_1 = I$ is the identity matrix of order $m$ and $A_j(x)$ are $M(m, \boldsymbol{R})$-valued smooth functions, where $M(m, \boldsymbol{R})$ denotes the set of all $m \times m$ real matrices. Let $L(x, \xi)$ be the symbol of $L(x, D)$:

$$L(x, \xi) = \sum_{j=1}^{n} A_j(x) \xi_j.$$

Our main concern is to study when we can symmetrize $L(x, \xi)$ smoothly. We write

$$L(x, \xi) = (\phi_j^i(x, \xi))$$

where $\phi_j^i(x, \xi)$ stands for the $(i, j)$-th entry of $L(x, \xi)$ which is a linear form in $\xi$.

DEFINITION 1.1. Let us denote

$$d(L(x, \cdot)) = \dim \operatorname{span}\{\phi_j^i(x, \cdot)\}.$$

We call $d(L(x, \cdot))$ the reduced dimension of $L$ at $x$. This is equal to the dimension of the linear subspace of $M(m, \boldsymbol{R})$ spanned by $A_1(x), A_2(x), \ldots, A_n(x)$.

DEFINITION 1.2. We say that $A \in M(m, \boldsymbol{R})$ is real diagonalizable if $A$ is diagonalizable and all eigenvalues of $A$ are real.

---

Received December 20, 1999.
Revised July 17, 2000.