TSUKUBA J. MATH.
Vol. 25 No. 1 (2001), 47–73

EXPLICIT STRUCTURES OF THREE-DIMENSIONAL HYPERSURFACE PURELY ELLIPTIC SINGULARITIES OF TYPE (0, 1)

By

Naohiro KANESAKA

Introduction

In this paper, we give an explicit description of a certain class of singularities of algebraic varieties of dimension greater than or equal to two using toric geometry. Singularities appearing in an algebraic variety which is a closed subset in an affine space $\boldsymbol{C}^n$ for some positive integer n defined by a regular function on $\boldsymbol{C}^n$ is called hypersurface singularities, which we will investigate in the following sections. Especially, our subject is investigating so-called hypersurface *purely elliptic singularities.*

Watanabe [18] introduced the notion of purely elliptic singularities. In two-dimensional case, the notion of purely elliptic singularities is equivalent to that of cusps and simple elliptic singularities. Cusps are characterized as normal two-dimensional singularities the exceptional sets of whose minimal resolutions are circles of rational curves and appears, for example, in Hilbert modular surfaces, while simple elliptic singularities are characterized as two-dimensional normal singularities the exceptional sets of whose minimal resolutions consist of non-singular elliptic curves. These two-dimensional purely elliptic singularities are much investigated by many researchers.

We already know due to Ishii, Watanabe and other researchers that in three-dimensional Gorenstein purely elliptic singularities, some analogies of two-dimensional cases hold. For example, Ishii-Watanabe [9] defined a simple $K3$ singularity to be a normal Gorenstein isolated singularity of which the exceptional set of $\boldsymbol{Q}$-factorial terminal modification consists of a normal $K3$ surface, of course which is an analogy of simple elliptic singularities in two-dimensional cases. And simple $K3$ singularities are three-dimensional purely elliptic singularities.

1991 *Mathematics Subject Classification.* Primary 32S25; Secondary 14B05, 32S05.
Received July 27, 1999.
Revised January 10, 2001.