TSUKUBA J. MATH.
Vol. 24 No. 1 (2000), 197–207

TOPOLOGICAL LATTICES $C_k(X)$ AND $C_p(X)$: EMBEDDINGS AND ISOMORPHISMS

By

Toshiji TERADA

Abstract. For a Tychonoff space X, the topological lattices $C_k(X)$ and $C_p(X)$ of all real-valued continuous functions on X endowed respectively with the compact-open topology and the topology of pointwise convergence are studied. It is proved that $C_k(X)$ and $C_k(Y)$ are isomorphic if and only if $C_p(X)$ and $C_p(X)$ are isomorphic if and only if X and Y are homeomorphic. It is also shown that $C_p(Y)$ is embedded in $C_p(X)$ as a topological sublattice if and only if Y is a continuous image of a cozero-set of X.

1. Introduction

All spaces considered here are Tychonoff topological spaces. For a space X, the set of all real-valued continuous functions on X is denoted by $C(X)$. The subset of $C(X)$ consisting of bounded functions is denoted by $C^*(X)$. These sets can be regarded as lattices with respect to the order: $f \leq g$ if and only if $f(x) \leq g(x)$ at every point $x \in X$. Ring structures on $C(X)$ and $C^*(X)$ are also defined as usual and have been studied extensively. In case topological spaces are assumed to be compact, the following are famous.

KAPLANSKY THEOREM [4]. For compact spaces X and Y, if there is a lattice isomorphism between $C(X)$ and $C(Y)$, then X and Y are homeomorphic.

GELFAND-KOLMOGOROFF THEOREM [2]. For compact spaces X and Y, if there is a ring isomorphism between $C(X)$ and $C(Y)$, then X and Y are homeomorphic.

The Gelfand-Kolmogoroff theorem is considered as a corollary of the Kaplansky theorem since every ring isomorphism between function spaces is a

Received April 26, 1999