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ON EQUATIONS OF THE TYPE $Au = g(x, u, Du)$ WITH DEGENERATE AND NONLINEAR BOUNDARY CONDITIONS

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1. Introduction and Main Results

Let $\Omega \subset \boldsymbol{R}^n$ be a bounded domain with C^∞ boundary $\partial\Omega$. Let

$$Au(x) = -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(\sum_{j=1}^{n} a_{ij}(x)\frac{\partial u}{\partial x_j}(x)\right) + c(x)u(x)$$

be a second order elliptic differential operator with real C^∞ functions a_{ij}, c on $\overline{\Omega}$ satisfying the following properties:

(p1) $a_{ij}(x) = a_{ji}(x),\ i, j = 1, \dots, n, x \in \overline{\Omega}$.

(p2) There exists a positive constant C_0 such that for all $x \in \overline{\Omega}$ and all $\xi \in \boldsymbol{R}^n$

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq C_0|\xi|^2.$$

(p3) $c(x) \geq 0$ in $\overline{\Omega}$.

Let Du be the gradient of u. We consider the following class of *degenerate* boundary value problems for semilinear second order elliptic differential operators

$$\text{(P)} \qquad Au = g(x, u, Du) \text{ in } \Omega, \quad Bu = a\frac{\partial u}{\partial \nu} + bu = \varphi \text{ on } \partial\Omega$$

in the framework of Sobolev spaces $W_p^2(\Omega)$ with $p > n$, where B is a degenerate boundary operator. Let us remark that

$$W_p^2(\Omega) \hookrightarrow C^1(\overline{\Omega}) \quad \text{if } p > n,$$

where $\hookrightarrow$ denotes the continuous embedding. Here:

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