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# ON SHIMURA LIFTING OF MODULAR FORMS

By

Shigeaki TSUYUMINE

**0.** Let $k, N \in \boldsymbol{N}$, $4|N$. Let $M_{k+1/2}(N, \chi_0)$ denote the space of modular forms for $\Gamma_0(N)$ of weight $k+1/2$ with a character $\chi_0 \pmod{N}$. Lifting maps of cusp forms in $M_{k+1/2}(N, \chi_0)$ to modular forms of integral weight was first studied by Shimura [7] and later by Niwa [4]. The domain of the map is extended to $M_{k+1/2}(N, \chi_0)$ by van Asch [1] in case that $\chi_0$ is real and $N = 4p$ for $p$ prime, and by Pei [5] in case that $\chi_0$ is real and $N/4$ is square-free. In the present paper we consider the lifting map without any condition on $N$ and $\chi_0$, and extend the domain of the map to $M_{k+1/2}(N, \chi_0)$ for $k \geq 2$.

To show the assertion, we take some specific modular forms in $M_{k+1/2}(N, \chi_0)$ which together with cusp forms, span $M_{k+1/2}(N, \chi_0)$. Further we construct their liftings explicitly. It proves our main result. It may be expected to have further application to study of special values of $L$-series of Hecke eigen cusp forms, as in Zagier [9], Kohnen-Zagier [3] where the lifting of some particular modular forms plays an important role.

**1.** We denote by $\boldsymbol{N}, \boldsymbol{Z}, \boldsymbol{C}$, the set of natural numbers, the ring of integers and the complex number field respectively. For a prime $p \in \boldsymbol{N}, v_p$ denotes the $p$-adic valuation. For $N \in \boldsymbol{N}, (\boldsymbol{Z}/N)^*$ denotes the group of Dirichlet characters $(\operatorname{mod} N)$. When $N = 1$, the group is consisting of a constant 1. The identity element of $(\boldsymbol{Z}/N)^*$ is denoted by $1_N$. A group consisting of invertible elements in $\boldsymbol{Z}/N$ is denoted by $(\boldsymbol{Z}/N)^\times$. If $\chi \in (\boldsymbol{Z}/N)^*$ and $e \in \boldsymbol{N}$, the $\chi^{(e)}$ denotes a character $(\operatorname{mod} eN)$ obtained by $\chi^{(e)}(d) = \chi(d)$ $((d, e) = 1)$, $0$ $((d, e) \neq 1)$. In case that all prime factors of $e$ appear as factors of $N$, then $\chi^{(e)}$ is equal to $\chi$. For $a \in \boldsymbol{Z}$ and for an odd $b \in \boldsymbol{N}$, $(a/b)$ denotes the Jacobi-Legendre symbol where it is 0 if $(a, b) \neq 1$. If $D$ is a discriminant of a quadratic field, then $\chi_D$ denote the Kronecker-Jacobi-Legendre symbol. We put $\chi_D = 1$ for $D = 1$.

Let $\mathfrak{H}$ denote the upper-half plane $\{z \in \boldsymbol{C} \mid \operatorname{Im} z > 0\}$. The group $SL_2(\boldsymbol{Z})$ acts on $\mathfrak{H}$ by the usual modular transformation sending $z \in \mathfrak{H}$ to $Mz = (az+b)/$

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