TSUKUBA J. MATH.
Vol. 23 No. 3 (1999), 419–434

A POWER SERIES SATISFYING A CERTAIN FUNCTIONAL EQUATION

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1. Introduction

In this paper, we are concerned with an enumeration of rooted trees. We consider isomers of chain saturated mono-hydroxy alcohols, that is to say, having no double, triple bonds and cyclic structure. Since the carbon atom has a valency of four and the hydrogen atom a valency of one, the structural formulas of these isomers form ternary rooted trees. For example, the following figures indicates that two isomers of propyl-alcohols:

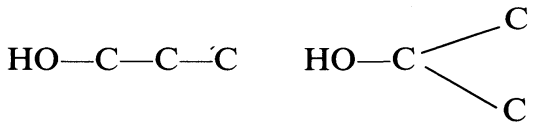

Figure 1. Propyl-alcohol

In this figure, we regard that the remaining valencies of carbon atoms are bonded with hydrogen atoms. Let $C(n)$ be the number of the isomers of the alcohols containing n carbon atoms. We define $C(0) = 1$. Clearly $C(n) \geq 1$ $(n \geq 0)$ is nondecreasing. We define a power series $g(z)$ by

$$g(z) = \sum_{n \geq 0} C(n) z^n = 1 + z + z^2 + 2z^3 + 4z^4 + 8z^5 + 17z^6 + \cdots, \tag{1}$$

which satisfies the functional equation:

$$g(z) = 1 + \frac{z}{6}(g(z)^3 + 3g(z)g(z^2) + 2g(z^3)) \tag{2}$$

(cf. Temperley [5], Polya [4]). Regarding the alcohol enumeration problem considered in (2) Polya concluded that the number of isomeric hydrocarbons of

Received August 18, 1997
Revised June 7, 1999