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KLEIN'S SURFACE OF GENUS THREE AND ASSOCIATED THETA CONSTANTS

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Introduction

Klein's surface of genus three is the closed Riemann surface R defined by the equation

$$y^{7}=x(x-1)^{2}.$$

This surface is famous because the conformal automorphism group $Aut(R)$ has order $168=84(3-1)$, the maximum possible.

In 1970, Rauch and Lewittes wrote the beautiful paper [4] in which they found the period matrix of Klein's surface. It is difficult to determine the period matrix of a non-hyperelliptic Riemann surface of genus $g\geq 3$ explicitly.

Klein originally obtained his surface R in the form of the upper half-plane identified under the principal congruence subgroup of level seven, $\Gamma(7)$, of the modular group Γ. Rauch and Lewittes represented the surface as the unit circle uniformization of R. They found a canonical homology basis expertly and they had the matrix representations of the generators of the automorphism group $Aut(R)$. In order to find the period matrix with respect to this canonical homology basis they used these matrix representations.

The aim of our article is to decide the proportionalities of associated theta constants of Klein's surface.

In our article we represent R as a covering surface over P^{1}. Let T be a conformal automorphim of order 7 and let $\langle T\rangle$ be the cyclic group generated by T and $R/\langle T\rangle$ the surface obtained by identifying the equivalent points on R under $\langle T\rangle$. Then $R/\langle T\rangle$ becomes P^{1} conformally and so R is considered as a 7-sheet covering surface of $R/\langle T\rangle\cong P^{1}$.

In the first section we calculate the period matrix of R by the different method from that of Rauch and Lewittes. First we choose a basis of the space

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