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REAL HYPERSURFACES IN A COMPLEX HYPERBOLIC SPACE WITH THREE CONSTANT PRINCIPAL CURVATURES

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1. Introduction

Let $H_n(\boldsymbol{C})$ be a complex hyperbolic space of complex dimension n (≥ 2) with the metric of constant holomorphic sectional curvature -4 and M be a real hypersurface in $H_n(\boldsymbol{C})$ with the induced metric. We denote by $\tilde{J}$ the natural complex structure of $H_n(\boldsymbol{C})$.

S. Montiel [4] gave the following classification theorem.

THEOREM. *If M is a connected real hypersurface of $H_n(\boldsymbol{C})$ $(n \geq 3)$ with two distinct constant principal curvatures, then M is holomorphic congruent to an open part of one of the following real hypersurfaces of $H_n(\boldsymbol{C})$: a geodesic hypersphere in $H_n(\boldsymbol{C})$; a tube around $H_{n-1}(\boldsymbol{C})$ in $H_n(\boldsymbol{C})$; a tube of radius $ln(2+\sqrt{3})$ around $H_n(\boldsymbol{R})$ in $H_n(\boldsymbol{C})$; a horosphere in $H_n(\boldsymbol{C})$.*

Moreover, J. Berndt [1] classified all real hypersurfaces with constant principal curvatures in $H_n(\boldsymbol{C})$ under the assumption:

(C) The structure vector field is principal.

In this paper we prove that Berndt's theorem holds without the condition (C) for the case where the number of constant principal curvature is three and $n \geq 3$. More precisely,

MAIN THEOREM. *Let M is a connected real hypersurface in $H_n(\boldsymbol{C})$ $(n \geq 3)$ with three distinct constant principal curvatures. Then M is holomorphic congruent to an open part of one of the following hypersurfaces:*

(a) *a tube of radius $r \in \boldsymbol{R}^+$ around $H_k(\boldsymbol{C})$ for a $k \in \{1, \dots, n-2\}$,*

(b) *a tube of radius $r \in \boldsymbol{R}^+ \setminus \{ln(2+\sqrt{3})\}$ around $H_n(\boldsymbol{R})$.*

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