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WELL-POSEDNESS OF CAUCHY PROBLEMS FOR LINEAR EVOLUTION OPERATORS WITH TIME DEPENDENT COEFFICIENTS

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Introduction

Let

$$A(t, D_t, D_x) = D_t^m + a_1(t, D_x)D_t^{m-1} + \cdots + a_m(t, D_x)$$

be a linear partial differential operator, where $a_j(t, \xi)$ is a $\mathscr{B}^\infty(0, T)$-function of t with a parameter $\xi (\in R^n)$ satisfying

$$\sup_{0<t<T}|D_t^r a_j(t, \xi)| \leqq C_r(1 + |\xi|)^{p(r)} \quad (r = 0, 1, 2, \dots, \ \ \xi \in R^n),$$

where $C_r > 0$ and $p(r) \geqq 0$, and consider the Cauchy problem (P):

$$\begin{cases} A(t, D_t, D_x)u = f(t, x) & \text{in } \{0 < t < T, x \in R^n\}, \\ D_t^j u = g_j(x) \ \ (j = 0, 1, \dots, m-1) & \text{on } \{t = 0, x \in R^n\}. \end{cases}$$

Under what conditions is (P) well-posed in $H^\infty(R^n)$? The answer must be thought very easy, because the problem (P) can be reduced to the Cauchy problem of ordinary differential equations $(P^\wedge)$:

$$\begin{cases} A(t, D_t, \xi)u^\wedge = f^\wedge(t, \xi) & \text{in } \{0 < t < T\}, \\ D_t^j u^\wedge = g_j^\wedge(\xi) \ \ (j = 0, 1, \dots, m-1) & \text{on } \{t = 0\}, \end{cases}$$

where $f^\wedge$ is the Fourier transform of f with respect to x. But it is not so easy, because we have not explicit solutions in general. Of course, if $a_j(t, \xi)$ is independent of t, it is well known that (P) is well-posed in $H^\infty(R^n)$ iff there exists $C > 0$ such that

$$\operatorname{Im} \tau_j(\xi) \geqq -C \log|\xi| \quad (|\xi| \geqq 2) \ \ (j = 1, 2, \dots, m)$$

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