TSUKUBA J. MATH.
Vol. 23 No. 2 (1999), 235–243

REAL HYPERSURFACES OF A COMPLEX PROJECTIVE SPACE

By

Syuji NAKAJIMA

1. Introduction

Let $P_n(C)$ be an n-dimensional complex projective space with Fubini-Study metric of constant holomorphic sectional curvature 4. Typical examples of real hypersurface in $P_n(C)$ are homogeneous ones. R. Takagi ([8]) showed that all homogeneous real hypersurfaces in $P_n(C)$ are realized as the tubes of constant radius over compact Hermitian symmetric spaces of rank 1 or 2. Namely, he proved the following

THEOREM 1.1. *Let M be a homogeneous real hypersurface of $P_n(C)$. Then M is locally congruent to one of the following*:

(A_1) *a geodesic hypersphere (that is, a tube over a hyperplane $P_{n-1}(C)$)*,
(A_2) *a tube over a totally geodesic $P_k(C)$ $(1 \leq k \leq n-2)$*,
(B) *a tube over a complex quadric Q_{n-1}*,
(C) *a tube over $P_1(C) \times P_{(n-1)/2}(C)$ and $n(\geq 5)$ is odd*,
(D) *a tube over a complex Grassmann $G_{2,5}(C)$ and $n = 9$*,
(E) *a tube over a Hermitian symmetric space $SO(10)/U(5)$ and $n = 15$*.

On the other hand, many differential geometers have studied real hypersurfaces in $P_n(C)$ by making use of the almost contact metric structure induced from the complex structure J of $P_n(C)$ (see, §2). It is well-known that there does not exist a real hypersurface with parallel second fundamental tensor of $P_n(C)$. Moreover Hamada ([2]) showed that there does not exist a real hypersurface with recurrent second fundamental tensor A of $P_n(C)$, i.e., there exists a 1-form α such that $\nabla A = A \otimes \alpha$. In this paper we consider the weaker condition.

The second fundamental tensor A is called a birecurrent second fundamental tensor if there exists a covariant tensor field α of order 2 such that $\nabla^2 A = A \otimes \alpha$.

Received April 14, 1998.
Revised December 21, 1998.