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A SHORT INTERVALS RESULT FOR $2n$-TWIN PRIMES IN ARITHMETIC PROGRESSIONS

By

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1. Introduction

In 1937 I. M. Vinogradov [11] proved that for every sufficiently large odd integer N the equation

$$N = p_1 + p_2 + p_3 \tag{1}$$

has solutions in prime numbers p_1, p_2, p_3. Vinogradov's estimate for linear trigonometric sums over primes enabled Chudakov, Estermann and Van der Corput (for example, see [1]) to prove in 1937 that almost all even integers $2n$ can be written as a sum of two primes (Goldbach's problem):

$$2n = p_1 + p_2. \tag{2}$$

The arguments used in the proof of (2) show that a similar result holds for the $2n$-twin primes problem (for example, see [3]):

$$2n = p_1 - p_2. \tag{3}$$

Zulauf [12], [13] obtained asymptotic formulas for the number of the prime solutions of the equations (1) and (2) with $p_1 \equiv l \,(\mathrm{mod}\, k)$, $(l,k) = 1$. Zulauf's formulas hold uniformly for $k \le L^D$, where $L = \log N$ and $D > 0$ is some fixed constant. Recently Tolev [9] established the following result. Let us consider integers k, l such that $(k,l) = 1$ and denote

$$J_{k,l}(N) = \sum_{\substack{p_1+p_2+p_3=N \\ p_1 \equiv l \,(\mathrm{mod}\, k)}} \log p_1 \log p_2 \log p_3.$$

Then, for every $A > 0$ there exists $B = B(A) > 0$ such that

$$\sum_{k \le \sqrt{N} L^{-B}} \max_{(l,k)=1} \left| J_{k,l}(N) - \frac{N^2}{2\varphi(k)} \mathfrak{S}_{k,l}(N) \right| \ll N^2 L^{-A}. \tag{4}$$

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