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NOTE ON MACAULAY SEMIGROUPS

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Almost all of ideal theory of a commutative ring R concerns properties of ideals of R with respect to the multiplication "$\times$" on R. Abandoning the addition "$+$" on R we extract the multiplication on R. Then we have the idea of the algebraic system S of a semigroup. We denote the operation on S by addition. S is called a grading monoid. Concretely, a submonoid S of a torsion-free abelian (additive) group is called a grading monoid (or a g-monoid). Many terms in commutative ring theory are defined analogously for S. For example, a non-empty subset I of S is called an ideal of S if $S + I \subset I$. Let I be an ideal of S with $I \subsetneqq S$. If $s_1 + s_2 \in I$ (for $s_1, s_2 \in S$) implies $s_1 \in I$ or $s_2 \in I$, then I is called a prime ideal of S. If there exists an element $s \in S$ such that $I = S + s$, then I is called a principal ideal of S. The group $q(S) = \{s_1 - s_2 \mid s_1, s_2 \in S\}$ is called the quotient group of S. A subsemigroup of $q(S)$ containing S is called an oversemigroup of S. Let Γ be a totally ordered abelian (additive) group. A mapping v of a torsion-free abelian group G onto Γ is called a valuation on G if $v(x + y) = v(x) + v(y)$ for all $x, y \in G$. Then v is called a Γ-valued valuation on G. The subsemigroup $\{x \in G \mid v(x) \geq 0\}$ of G is called the valuation semigroup of G associated with v. A $\boldsymbol{Z}$-valued valuation is called a discrete valuation of rank 1. The valuation semigroup associated with a discrete valuation of rank 1 is called a discrete valuation semigroup of rank 1. An element x of an extension semigroup T of S is called integral over S if $nx \in S$ for some $n \in \boldsymbol{N}$. Let $\bar{S}$ be the set of all integral elements of $q(S)$ over S. Then $\bar{S}$ is an oversemigroup of S, and is called the integral closure of S. If $\bar{S} = S$, then S is called an integrally closed semigroup (or a normal semigroup). An ideal I of S is called a cancellation ideal of S if $I + J_1 = I + J_2$ (for ideals J_1, J_2 of S) implies $J_1 = J_2$. The maximum number n so that there exists a chain $P_1 \subsetneqq P_2 \subsetneqq \cdots \subsetneqq P_n (\subsetneqq S)$ of prime ideals of S is called the dimension of S. Many propositions for commutative rings are known to hold for S. The author conjectures that almost all propositions of multi-

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