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A NOTE ON KAEHLERIAN METRICS WITH CERTAIN PROPERTY FOR ∇RIC

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0. Introduction

Let M^{2n} be a $2n$ real dimensional Kaehlerian manifold, whose complex structure is given by a parallel tensor field $F = (F_j^i)$ satisfying

$$F_j^r F_r^i = \delta_j^i, \quad g_{ji} F_\ell^j F_k^i = g_{\ell k},$$

where $g = (g_{ji})$ is the Riemannian metric tensor of M^{2n}. Let $K = (K_{kji}^h)$ be the Bochner curvature tensor and $\hat{K} = (K_{kji})$ a tensor given by

$$\begin{aligned} K_{kji} &= \nabla_k R_{ji} - \nabla_j R_{ki} \\ &\quad + \frac{1}{4(n+1)} (g_{ki}\delta_j^h - g_{ji}\delta_k^h + F_{ki}F_j^h - F_{ji}F_k^h + 2F_{kj}F_i^h) r_h, \end{aligned} \tag{0.1}$$

where $R = (R_{kji}^h)$ is the Riemannian curvature tensor, $\mathrm{Ric} = (R_{ji}) = (R_{kji}^k)$ the Ricci tensor, and $r = R_k^k$ the scalar curvature.

Let us consider the condition

$$\nabla_k R_{ji} = \frac{1}{4(n+1)} (2r_k g_{ji} + r_j g_{ki} + r_i g_{kj} + \tilde{r}_j F_{ik} + \tilde{r}_i F_{jk}), \tag{$\sharp$}$$

where $\tilde{r}_j = F_j^h r_h$ and $r_j = \nabla_j r$. This condition gives a necessary and sufficient condition for equality in the inequality

$$\frac{1}{m+1} |dr|^2 \leqq |\nabla \mathrm{Ric}|^2$$

which was proved in [2].

If M^{2n} satisfies ($\sharp$) $\hat{K}$ vanishes [2]. But the converse is not true. The example of metric satisfying ($\sharp$) is unknown, except the case where r is constant.

On the other hand, if M^{2n} is Bochner-flat, i.e. $K = 0$, then $\hat{K}$ vanishes. The

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