TSUKUBA J. MATH.
Vol. 23 No. 1 (1999), 113–130

ON CURVATURE PROPERTIES OF CERTAIN GENERALIZED ROBERTSON-WALKER SPACETIMES

Dedicated to the memory of Professor Dr. Georgii Ionovich Kruchkovich

By

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1. Introduction

The warped product $\overline{M} \times_F N$, of a 1-dimensional manifold $(\overline{M}, \bar{g}), \bar{g}_{11} = -1$, with a warping function F and a 3-dimensional Riemannian manifold $(N, \tilde{g})$ is said to be a generalized Robertson-Walker spacetime (cf. [2], [32]). In particular, when the manifold $(N, \tilde{g})$ is a Riemannian space of constant curvature, the warped product $\overline{M} \times_F N$ is called a Robertson-Walker spacetime. In [11] it was shown that at every point of a generalized Robertson-Walker spacetime $\overline{M} \times_F N$ the following condition is satisfied:

$(*)_1$ the tensors $R \cdot R - Q(S, R)$ and $Q(g, C)$ are linearly dependent.

This condition is equivalent to the relation

$$R \cdot R - Q(S, R) = L_1 Q(g, C) \tag{1}$$

on the set $\mathcal{U}_C$ consisting of all points of the manifold $\overline{M} \times_F N$ at which its Weyl tensor C is non-zero, where L_1 is a certain function on $\mathcal{U}_C$. For precise definitions of the symbols used, we refer to the Sections 2 and 3. $(*)_1$ is a curvature condition of pseudosymmetry type. In this paper we will investigate generalized Robertson-Walker spacetimes realizing a condition of pseudosymmetry type introduced in [25]. Namely, semi-Riemannian manifolds (M, g), $n \geq 4$, fulfilling at every point of M the following condition

$(*)$ the tensors $R \cdot C$ and $Q(S, C)$ are linearly dependent.

were considered in [25]. This condition is equivalent to the relation

$$R \cdot C = LQ(S, C) \tag{2}$$

Received January 20, 1998
Revised June 2, 1998