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CONSTANT MEAN CURVATURE SURFACES WITH BOUNDARY IN EUCLIDEAN THREE-SPACE

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1. Introduction and Statements of Results

The study of the structure of the space of constant mean curvature compact surfaces with boundary in Euclidean space $\boldsymbol{R}^3$ is the focus of a number of authors. Even in the simplest case, when the boundary is a circle, little is known. For example, if the radius of the circle is $r > 0$, a necessary condition due Heinz [5] about the mean curvature H is that $|H| \leq 1/r$. In this case, the only known examples are the planar disc ($H = 0$), the two corresponding spherical caps of radius $1/|H|$ if $H \neq 0$, and some non-embedded examples of Kapouleas [8] with genus bigger than two. Also, if the immersion is an embedding, Koiso [9] proved that if the surface is included in one of the two halfspaces determined by the plane containing the circle, then the surface inherits the symmetries of its boundary, and so, the surface is a spherical cap or a planar disc. Recently, Alías, Palmer and the author have proved that the umbilical examples are the only constant mean curvature stable discs with boundary a circle [1].

We will abbreviate a constant mean curvature compact (connected) surface immersed in $\boldsymbol{R}^3$ by *cmc surface* and when we want to emphasize the value H of the mean curvature we will say *H-cmc surface*. Let Γ be a Jordan curve in $\boldsymbol{R}^3$. We say that Σ is a cmc surface *with boundary* Γ when the immersion $\phi : \Sigma \to \boldsymbol{R}^3$ maps the boundary of Σ, $\partial\Sigma$, onto Γ diffeomorphically. Remark that if $H = 0$, the minimal surface Σ is contained in the convex hull of Γ, and hence, if the boundary Γ is planar, the surface is included in the boundary plane.

When the boundary Γ is a circle of radius $r > 0$, Brito and Earp [3] characterized the equality $|H| = 1/r$ in the above Heinz condition, that is, they proved that the hemisphere is the only $(1/r)$-cmc surface bounded by the circle Γ. This was proved by using a kind of flux formula and the fact that umbilical

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