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THE CAUCHY PROBLEM FOR WEAKLY HYPERBOLIC EQUATIONS OF SECOND ORDER

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§1. Introduction

In this article we shall study the problem of local existence of C^{∞} solutions to the following semilinear Cauchy problem on $[0, T] \times \boldsymbol{R}_x^n (T > 0)$

$$L(t, x, \partial_t, \partial_x) u(t, x) = f(t, x, u), \tag{1.1}$$

$$u(0, x) = u_0(x), u_t(0, x) = u_1(x), \tag{1.2}$$

where

$$\begin{aligned} L(t, x, \partial_t, \partial_x) &= \partial_t^2 - a_1(t) \sum_{j,k=1}^{n} a_{jk} \{a_0(x) \partial_{x_j} \partial_{x_k} + (\partial_{x_j} a_0(x)) \partial_{x_k}\} \\ &\quad - \sum_{j=1}^{n} b_j(t, x) \partial_{x_j} - c(t, x) - d(t, x) \partial_t, \\ &= \partial_t^2 - a^{\sharp}(t, x, \partial_x) - b(t, x, \partial_x) - c(t, x) - d(t, x) \partial_t. \end{aligned}$$

Throughout the present article we assume that $0 < C_a \le a_0(x) \in \mathfrak{B}^{\infty}(\boldsymbol{R}^n)$, $q(\xi) = \sum_{j,k=1}^{n} a_{jk} \xi_j \xi_k \ge 0$ (a_{jk} is a real constant, $a_{jk} = a_{kj}$) for all $\xi \in \boldsymbol{R}^n$ and that $0 \le a_1(t) \in C^{\infty}([0, T])$ satisfies the condition below:

$$N = \operatorname{card}\{[p, q] \subset [0, T]; a_1'([p, q]) \subset \{0\}, a_1'(p - \varepsilon) a_1'(q + \varepsilon) < 0 (0 < \varepsilon \ll 1)\} \\ < \infty, \tag{1.3}$$

where card X means the cardinality of a set X, that is, the number N of the connected components of the sign-changing zero-set of a_1' on $[0, T]$ is finite. Moreover we impose that $b_j, c, d, f \in \mathfrak{B}^{\infty}$ (with $f([0, T], \boldsymbol{R}^n, 0) \subset \{0\}$) and that

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